

# Methods for Respecting Political Subdivisions, in Automated Redistricting

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## ARTICLE INFO

**Keywords:**  
gerrymandering  
preserving political subdivisions  
automated redistricting  
compactness

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## ABSTRACT

This project presents new methods for automated legislative districting. Recent efforts in this vein have focused primarily on the goals of equipopulation, contiguity, and spatial compactness, and on the implications for political parties. Lawmakers and lawyers have long recognized the additional objective of reducing the division of existing political communities. This project demonstrates that existing work can be adapted to capture that additional aim, using simple geometric manipulations of the underlying geometry. In short, scaling communities or their perimeters elevates jurisdictional divisions into physical space. This manipulation makes it possible to use existing optimization methods to minimize the division of communities. An empirical exercise demonstrates that the scaling approach provides an effective sliding scale for reducing the division of existing communities, in this case counties. The scaling does not affect the outcomes expected for the dominant political parties; the choice of scaling value is not a hidden partisan choice. The project thus extends the objectives that algorithms can be used to optimize.

Districting reform at the Federal level in the United States has long focused on the courts, but the Supreme Court's decision in *Rucho v. Common Cause* foreclosed the possibility of Federal, judicial relief. Congress has the explicit constitutional authority to regulate gerrymandering and a long history of doing so. This project seeks to revive and inform that approach.

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After three decades waffling over its role in gerrymandering reform, the Supreme Court ruled in *Rucho v. Common Cause* that the issue is non-justiciable. In his opinion for the majority, Chief Justice Roberts recognized the harm and the outrage of gerrymandering. He suggested that reformers working at the Federal level concentrate their energies on legislative avenues that are both constitutionally-sanctioned and have historically proven successful. (588 U.S., 2019) Legally, the US Congress could simply decide to outlaw partisan districting of Federal constituencies.

What might Federal reform look like? Past statutes required Congressional districts to be equipopulous, contiguous, and compact. Existing software can be used to optimize statewide maps for these three objectives. However, many states require – and many mapmakers desire – that district boundaries respect existing political subdivisions or communities of interest. Along with constitutional and statutory requirements for and a moral interest in guaranteeing minority representation, this interest in preserving formal and informal communities is the final “traditional districting principle.” Nineteen state constitutions require it (Levitt, 2020), the Supreme Court has routinely recognized it (*Davis v. Bandemer*, 478 U.S. 109, 1986; *Miller v. Johnson*, 515 U.S. 900, 1995; *Reynolds v. Sims*, 377 U.S. 533, 1964; *Shaw v. Reno*, 509 U.S. 630, 1993), and it is taken for granted in the academic and legal literatures. (Altman, 1998; Chen and Rodden, 2015; Stephanopoulos and McGhee, 2015) This project describes how existing work on spatial compactness

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can be adapted to evaluate the preservation of existing political communities, and minimize their division in automated contexts.

Equipopulation and contiguity have unambiguous and undisputed mathematical definitions. Compactness and respect for communities of interest do not. Compactness suffers from a confusion of possibilities. Dozens of competing formulae quantify the relationships of perimeters to the edge, the spatial distribution of voters, or districts' similarities to a variety of derived reference shapes. (Hofeller and Grofman, 1990; Niemi et al., 1990) However, recent work has shown that in an automated, non-partisan context, these many definitions ultimately lead to consistent political outcomes. (Saxon, 2020) The choice among methods affects the shapes of the districts, but not partisan representation.

But what of a respect for "existing political subdivisions?" Unlike compactness, equipopulation, or contiguity, there are neither recognized formulae for evaluating nor algorithms for minimizing the divisions of existing spatial regimes. How might respect for communities be defined, and how might it be incorporated in automated methods?

The core idea of this paper is that regionalization algorithms designed to optimize spatial compactness can often be adapted trivially to account for existing political boundaries. This is accomplished by manipulating the cost of connections between regimes, by elevating jurisdictional structures into physical space. Two examples illustrate the point. If the perimeter is the cost, one re-scales the cost of perimeters along political (or physical, or cultural) boundaries to make it "cheaper" in the optimization to divide regions at those divisions. Alternatively, if the cost is a distance between voters, one shrinks each jurisdiction towards its own centroid, so that there are artificially large distances between jurisdictions. Either strategy provides a sliding scale between spatial compactness and respect for jurisdictions.

This paper proceeds in four parts. I first review the legal history and current context. I then describe three definitions of compactness and show how they can be modified in an optimization framework to prioritize the preservation of existing political subdivisions. I briefly define a metric to quantify split communities. Finally, I generate maps for three recently litigated states: Maryland, North Carolina, and Pennsylvania. I use these maps to show that the methods described in fact reduce the divisions of counties, and evaluate the implications for the representation of the political parties. I conclude with notes on directions for continued research.

## 1. The Legal, Political, and Algorithmic Context

The US Constitution grants tentative control over the form of (Federal) Congressional districts to state legislatures. Congress retains ultimate supervisory power however, over the "Times, Places, and Manners" of the elections of its members (Article 1, Section 4). The founders granted this authority to Congress to ensure:

1. that the states would not deprive the Federal government of its legitimacy by refusing to hold elections, but also
2. uniformity of representation in the face of potential perversions by state legislatures. James Madison foresaw that legislatures would manipulate delegations according to their partisan inclinations and opined that, without corrective Federal measures, this would result in "diversity [of regulations that] would be obviously unjust."

The Federal power was clearly understood at the time and it was debated vigorously in the state ratifying conventions. (Kurland, et al., 2000) At least seven of the thirteen original states directed that their delegations to Congress work to restrict the Federal oversight. (Cong. Globe, 1842) This did not happen.

For nearly a century, from 1842 until 1929, Congress used this authority to curb abuses by the legislatures. Through the first half of the nineteenth century, a number of states elected members of Congress

from at-large districts. This practice was widely understood to increase the power small states, whose Congressional delegations were uniform “blocs.” (Zagarri, 1987) After a heated debate, Congress outlawed the practice in 1842, by requiring that each member represent a single, *contiguous* district. (5 Stat. 491, 1842; Cong. Globe, 1842) In 1872, it stipulated further that districts be *equipopulus*, and this requirement was retained in 1882. (17 Stat. 28, 1872; 22 Stat. 5, 1882) In an effort to do away with “shoestring districts,” Congress required in 1901 that districts be *compact*. (Cong. Rec., 1901) These requirements persisted until 1929, when Congress’s attention was consumed by the long squabble over apportionment that arose from the botched 1920 Census.<sup>1</sup> (Cong. Rec., 1929) When Congress tried to reassert the requirements of equipopulation, contiguity, and compactness in 1967, the bill passed both houses but failed in conference. (Cong. Quarterly, 1968)

As Congress abdicated its power, the Supreme Court assumed it. Beginning with the “Redistricting Revolution” of the early 1960s, the Court radically redrew the line between legislative and judicial authority. In *Baker v. Carr* (369 U.S. 186, 1962), the Court ruled that justiciable questions are ones that, among other conditions, admit “clear and manageable standards.” Applying this new standard, the Court did away with malapportioned districts (of unequal populations) in just a few years.<sup>2</sup> In 1986 with *Davis v. Bandemer*, the Court extended its purview to the next perversion of the districting process: partisan gerrymandering. (478 U.S. 109, 1986) It ruled that though the justices could not agree on a manageable standard, they expected that one might ultimately be articulated. They therefore declined to provide relief but continued to invite cases on the issue for over thirty years. In this time, the Court distracted attention from the existing, legislative avenues. In 2019, the Court gave up the chase with *Rucho v. Common Cause*. Partisan gerrymandering is a real harm, but not a justiciable question.

What is to be done? In decades past, algorithms seemed to promise a solution to the influence of partisan actors on the districting process. Then-Governor Ronald Reagan opined in 1972 that “there is only one way to do reapportionment – feed into the computer all of the factors except political registration.” (Goff, 1972) Similarly, Justice William Brennan wrote in 1983 that, even at that time, computers “make it relatively simple to draw contiguous districts of equal population and at the same time to further whatever secondary goals the State has.” (Karcher v. Daggett, 462 U.S. 725, 1983) From the dawn of computation in the early 1960s, Vickrey, Weaver, Hess and others created moderately powerful algorithms for small cases. (Vickrey, 1961; Weaver and Hess, 1963) But as the scope and complexity of potential districting considerations became clear, later efforts lost steam. Altman et al. reviewed the potential of automated districting (2005) and developed Better Automated Redistricting (BARD) in 2008, but never demonstrated its functionality in practice. (Altman and McDonald, 2011)

The last several years have seen a resurgence of algorithms in districting work, but their application has been primarily descriptive or forensic rather than prescriptive and aspirational. Chen and Rodden (2013, 2015) and Cho, Liu, and Wang (2016; 2016) blazed the trail for “ensemble methods” for evaluating gerrymandering and demographic dynamics in legislative districting. Each developed algorithms to optimize maps according to one version of districts’ compactness, though neither ran these algorithms to convergence. These algorithms could then be used to generate distributions of maps, that served to contextualize enacted plans. Subsequent work has adopted Markov Chain Monte Carlo (MCMC) methods to similar end: to mea-

<sup>1</sup> A difficult winter in 1920 led to apparent under-counting of rural regions. Congress is constitutionally required to reapportion seats after each Decennial Census, but it failed to do so in 1922 due to objections by representatives from rural states. Congress as a whole therefore looked like it was shirking its duty. The focus in the 1929 debate was on how to deal with automatic reapportionment in future Censuses. Only two representatives engaged in the debate over the *forms* of districts, and the requirements appear to have disappeared with scant notice on January 11, 1929. The Supreme Court ruled in 1932 that, by failing to restate the conditions required in 1901 and 1911, Congress had annulled them. (Wood v. Broom, 287 U.S. 1, 1932)

<sup>2</sup>Unequal district populations did not die without a fight. In particular, US Senate Minority Leader Everett Dirksen fought hard against the requirement of *Reynolds v. Sims* that districts in the upper houses of state legislatures be equipopulous. He got 37 of 38 states to vote for a Constitutional Convention, before passing away in 1969. The fight died with him. (Smith, 2014)

sure whether enacted plans were outliers in their treatment of political parties. (Bangia et al., 2017; Chikina et al., 2017; Fifield et al., 2020) Among these projects, less attention has been directed towards contrasting maps optimized according to different explicit criteria. The distributions in those projects do effectively show that enacted maps are severe outliers from certain MCMC distributions but they often do *not* show the degree of discrepancy from the possible space of plans, optimized according to objectives that may, in many cases, legally include political factors. On these grounds, the existing distributions are often not quite the apposite, forensic counterfactual. Since any enacted map is optimized to certain criteria, the relevant comparison is emphatically *not* a random contiguous, equipopulous partition of the state.<sup>3</sup> The project by Saxon (2020), focused on implementing and contrasting various objectives, is unusual in contrasting districting objectives and algorithms.

This project evaluates the potential impacts, if Congress were to require algorithmically-optimized districts. There are two common objections to automated optimization procedures. The first is that the problem is simply not tractable – an assertion usually made with reference to the formal, computational complexity of the solution space (NP-hard). But this defeatist position is untenable. The proof is by counterexample: in addition to the software reviewed above, many other earnest attempts at algorithms and engineered solutions have proven successful, and these are quickly growing more sophisticated. (Levin and Friedler, 2019; Olson, 2017) Regionalization is a classic problem in geography, just as graph partitioning is in graph theory, and many performant algorithms are available. (Duque et al., 2011a,b, 2012; Guo, 2008; Guo and Wang, 2011) The optimal solution will never be found and validated – that is what the technical definition of NP-hard means – but very good solutions are in hand. What is needed is work on faster, subtler, and more-encompassing methods. This project extends the earlier work by Saxon, with additional districting objectives. The second objection, related to the first, is that automating solutions are far more complex than gerrymandering, and would entail simultaneously implementing and balancing a number of competing objectives. (Altman, 1997) This point is granted, but it should be a spur for civic technologists – not a call for retreat.

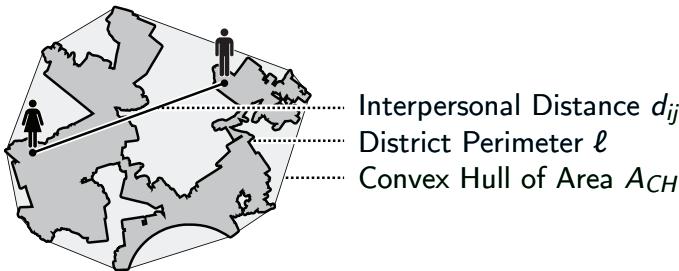
What are the objectives to be satisfied? The most obvious are the ones enacted by past Congresses, recognized by the Supreme Court, and required in state constitutions as “traditional districting principles.” These are equipopulation, contiguity, compactness, and a respect for existing political subdivisions. Along with these is a constitutional, statutory, and moral imperative to account for minority representation. Existing work provides methods for optimizing districts for equipopulation, contiguity, and compactness. This paper presents methods for minimizing the divisions of existing political subdivisions.

With the judicial pathway closed, many reformers have ignored Justice Roberts’ suggestion as facetious at best, and taken their fight to the states (indeed, they did so even before *Rucho*). Litigation at the state level requires clear counterfactual ensembles, able to account for a state’s full set of legitimate districting objectives. This project advances these methods. In Arizona and California, ballot measures successfully forced the states to district using independent commissions. But commissions must be charged with formally articulated objectives for the districting process. What should these objectives be and how may they be expressed? What are the political implications of these objectives and how may they be assessed? Although this paper does not focus on the avenues of commissions or state-level litigation, a quantitative understanding of districting methods and their impacts informs both of those approaches to reform.

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<sup>3</sup> Though the Supreme Court retreated in *Rucho* from judgements about the appropriateness of Federal Congressional districts, the forensic project remains useful in state-level litigation, since the methods can be used to show that political factors *had* to be included to obtain the extreme maps enacted.

**Figure 1:** Illustration of the quantities used to define the three compactness metrics: the interpersonal distance  $d_{ij}$ , district perimeter  $\ell$ , and convex hull area ratio  $A_{CH}$ . The illustration uses Pennsylvania's 7th Congressional District as enacted in 2012.



## 2. Objectives and Algorithms for Districts

This Section first presents several standard mathematical expressions for spatial compactness. The central contribution of the paper then follows: a suggestion for modifying geographic data to project political boundaries into physical space. Finally, it briefly reviews the automated methods used to generate districts.

### 2.1. Measuring Spatial Compactness

There are dozens of mathematical expressions for spatial compactness. (Niemi et al., 1990; Saxon, 2020) Three definitions are used in this paper, and illustrated in Figure 1.

#### *Interpersonal Distances / Power Diagrams*

The first metric is the average distance squared between co-constituents. If districts' populations  $N$  are aggregated in cells (Census tracts)  $i$  of population  $w_i$ , separated from each other by distances  $d_{ij}$ , then this can be written as  $\sum_i \sum_j w_i w_j d_{ij}^2 / N^2$ . Distance between co-constituents is “bad,” so lower averages are better.

Fryer and Holden (2011) have proven that this quantity is minimized by additively weighted power (or Voronoi) diagrams, which are relatively straightforward to implement. (A procedure is described below.) Power diagrams can be understood as defining regions by a reference point, and assigning areas in a plane to the region with the closest reference point. For additively weighted diagrams, each region also has an offset that determines its scale; this offset can be used to achieve balanced populations. Power diagrams result in clean, convex polygons that are intuitively appealing. However, district boundaries may divide communities. That is the behavior that this project aims to limit.

#### *Convex Hull Area Ratio*

A shape's *convex hull* is the smallest convex polygon that contains it. A compactness measure may be defined as the ratio between the area of the shape itself and that of its convex hull. Denoting the district's area by  $A$  and the area of its hull by  $A_{CH}$ , this is just  $A / A_{CH}$ . In this measure, an optimally compact shape is itself a convex polygon, and covers its convex hull. Unlike power diagrams, statewide maps that are compact according to the convex hull ratio may be long or distended.

#### *Isoperimeter Quotient / Polsby-Popper*

The isoperimeter quotient (IPQ) is the ratio of the district's area to a circle of equal perimeter. Call the length of the perimeter  $\ell$  and the area of the district  $A$  as above. A circle of circumference  $\ell$  has radius  $\ell / 2\pi$ , so the circle of perimeter equal to the district is  $\pi (\ell / 2\pi)^2 = \ell^2 / 4\pi$ . The IPQ is thus  $4\pi A / \ell^2$ . This measure is usually credited to [Polsby and Popper \(1991\)](#) in the districting literature, but Nagel used it to characterize shorelines as early as 1835. (See the discussion by [Frolov \(1975\)](#).)

While this method is appealing, the IPQ is sensitive primarily to the roughness of the perimeter, and is largely unresponsive to overall shape. Plans that maximize the IPQ are typically smooth but can be lumpy. Although perimeter is readily calculated with modern GIS tools, there are subtle definitional issues about the scale at which the perimeter is measured, since natural boundaries like shorelines and rivers often exhibit “fractal” behavior. In this project, perimeters between cells (Census tracts) are simplified at the largest scale that preserves the topology of Census tract edges and faces in the state.

## 2.2. Respecting Existing Communities Using Spatial Approaches

The aim of this paper is to point out that the measures above apply not only to evaluation and optimization of spatial compactness. They can also be used to assess and minimize the division of existing political subdivisions.

To do this, two scaling schemes can be applied. For the interpersonal distance and convex hull measures, one need only physically rescale spatial regimes towards their centroids. In the empirical example, the “existing political subdivisions” that I will consider are counties. The first row of Figure 2 illustrates the rescaling for Pennsylvania. The impact of this procedure is to make it more costly to cross between the scaled spaces. In the interpersonal distance approach, the distance among constituents is (artificially) reduced for residents of the same county. For the convex hull area ratio, the numerator (the total area of the fundamental cells) is reduced while the denominator (the area of the hull) must account for empty spaces when configurations cross between multiple regions/districts.

For the isoperimeter quotient, instead of scaling the areas one instead rescales the perimeters. Continuing the example with counties, one would reason that the “cost” of district borders ought to be lower along county lines. One makes it so, by manipulating the pre-calculated, physical perimeters between cells according their (co)membership in jurisdictions. This means that the area to perimeter ratio is higher (the denominator is smaller) if the perimeter adheres to the boundaries of the spatial regime (counties). For both IPQ and the areal scaling methods, I rescale by a linear factor that I will denote by  $S$ . Note that this scaling affects the lengths or distances, so areas scale as  $S^2$ .

Although I will apply these scaling methods to counties, it is worth emphasizing that they apply naturally to any set of nested spatial regimes. For example, the fundamental cells, that I will take as Census tracts, may be contained in neighborhoods or wards. These in turn would be in towns or cities, themselves within counties, that are perhaps assigned to regions of the state. For the areal scaling, one would first shrink the cells towards their neighborhood centroid, then towards their city centroid, and finally towards the county and region centroids – each potentially by a different factor. For the perimeter scaling, one would analogously down-weight the cost of edges along county, city, ward, or neighborhood lines. This applies equally to natural boundaries. For example, one could reduce or even eliminate the cost of perimeters between cells on opposite sides of a river, canyon, or mountain range. Taking things a step further, one could reevaluate or reallocate perimeters according to data on human behaviors (commuting ties or GPS mobility data) or the physical environment (number of streets crossing between cells).

To be clear, the compactness measures described above are by no means a “random” set. The strategies of scaling areas or perimeters are not always possible. For example, a common variant of the convex hull method is defined as the ratio of a district’s *population* to that of its convex hull, instead of the ratio of the areas. This population-based method is minimally responsive to the spatial scaling. Applying the areal scaling technique will separate the spatial regimes, but because this “gap” remains unpopulated, there is no penalty for crossing between regimes. The convex hull does not get more populous, the denominator does not increase, and the compactness does not fall as the spatial regimes are separated. By contrast, the spatial separation increases the cost of bridging regimes with the area ratio.

In addition, the scaling approach to communities is limited to spatial regimes that are already contiguous and somewhat compact. It does not effectively assemble geographically separated “communities of

interest.” For example, physical scaling is not an effective approach for uniting the dominantly-Hispanic neighborhoods of Austin with their ethnic counterparts in San Antonio (as Texas’s 35th aims to do, post 2010 Census), or even the two Hispanic communities of Chicago’s West Side (Illinois’s 4th). This raises two questions that I postpone to future work – first, how and why to define “communities of interest,” and second, how to incorporate them in algorithms. One possible answer lies in the power diagram method. In the present approach, spaces are scaled physically, but individual “distances” (dissimilarities) could be adapted to represent aspatial factors like income or ethnicity.

### 2.3. Automated Districting

Automated regionalization is performed using the *C4* software by [Saxon \(2020\)](#). This software provides general-purpose greedy optimization for district equipopulation and compactness, and enforces contiguity constraints using integer programming. A number of compactness “objectives” are implemented, including IPQ and the convex hull area ratio. The optimization proceeds cyclically over regions in a state, moving the single cell (here, a Census tract) with the highest score for improved equipopulation and spatial compactness, while preserving the contiguity of regions (Congressional districts). If no improvement is possible, no cell is moved. The algorithm terminates after a configurable number of cycles with no improvement. Initializing the algorithm with many seeds allows for an ensemble of high-quality maps.

For minimizing (squared) interpersonal distances, power diagrams, are implemented as a separate, three-step routine.

1. Regions  $r$  are defined by central points  $\mathbf{x}_r$  and scale offsets  $\lambda_r$ . At initialization, random points chosen and the scales are set to 0.
2. Census tracts of center  $\mathbf{x}_c$  are assigned to the region for which  $|\mathbf{x}_r - \mathbf{x}_c|^2 - \lambda_r^2$  is minimized.
3. Regions’ central points slowly move towards their centroids while their scales increase and decrease to equalize population.

Steps (2) and (3) repeat until population equality is achieved.

To apply this software to the present problem, it suffices to scale perimeters and polygons, before running the optimization. For the purposes of this paper, I use Census tracts as the fundamental cell. This can be motivated as a basic respect for local neighborhoods, since I enforce population equality (convergence) at the 2% level. This is clearly looser than the “no *de minimis* level” articulated by the Supreme Court ([Karcher v. Daggett, 462 U.S. 725, 1983](#)), but the context is very different: explicit Congressional reform and “Court curbing.” It is worth noting that Congress is the judge of its own elections (Article 1, Section 5), and that the last attempt at reform, in 1967, considered population deviations as large as 10%. ([Cong. Quarterly, 1968](#))

## 3. Evaluating Simulated Districts

After generating statewide districting plans, I evaluate two questions. First, does the “scaling” approach effectively reduce the division of existing political communities? Second, does it affect political outcomes? I contrast the maps simulated with different scaling values, with enacted ones.

This Section presents the metrics used for these results.

### 3.1. Split Communities

A common way of evaluating how well a districting plan respects existing communities is to count the number of times those communities are split between districts. This method is insensitive to variation in how “grievously” the community is divided. It may be worse to split a county in two than to shave a bit from its edge. Naïve counts also ignore the fact that some communities exceed the scale of a legislative district and *must* be divided. (However, because this will be true for *any* potential plan, comparative split counts remain valid.)

I define a measure of community splits as the likelihood of two residents of a community having the same legislator. Formally, I define this likelihood by dividing the population of shared (intersecting) county residence and Congressional representation by its maximum value. The denominator is then the lesser of the community (county) population and that of the legislative district. I will denote the legislative district of individual  $i$  by  $\text{LD}(i)$ , his or her community by  $\text{comm}(i)$ , and their intersection by  $\text{LD}(i) \cap \text{comm}(i)$ . Populations are the function,  $p(\cdot)$ . The “shared legislator” value for constituent  $i$  can then be written

$$\mathcal{L}_i = p(\text{LD}(i) \cap \text{comm}(i)) / \min(p(\text{LD}(i)), p(\text{comm}(i))).$$

As defined, this measure is bounded on  $[0, 1]$ . I focus on the statewide average,  $\bar{\mathcal{L}}$ . Higher values indicate that co-residents of a community (county) are more likely to be assigned to the same legislator.

This is one definition among many that could be defined, and over which reasoned debate might prove useful. It is *not* the aim of this paper to justify an optimal metric of community splits, from first principles or at a deep level. Rather, the intent is to define a reasonable benchmark for evaluating whether the spatial manipulations already described in fact accomplish their goal.

### 3.2. Partisanship

Partisan composition of simulated districts is evaluated by aggregating precinct-level returns from presidential elections, as assembled by [Saxon \(2019\)](#). The availability of granular, georeferenced returns differs among the states studied. Returns for Pennsylvania (2000–2012) and Maryland (2008) are originally drawn from [Ansolabehere et al. \(2015\)](#). Maryland data for 2016 are directly from the state. ([Maryland State Board of Elections, 2016](#)) Returns and precinct boundaries for North Carolina (2012, 2016) are from the state. ([North Carolina State Board of Elections, 2012, 2013, 2016a,b](#))

For each of the results that follow, the number of seats expected per-party is averaged across the available elections.

## 4. Results

For each value of the scaling parameter in  $S \in \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$  and for each of Maryland, Pennsylvania, and North Carolina, I use the *C4* software to generate 500 statewide plans using the IPQ and convex hull ratio methods, and 2500 plans with the power diagram method. Solutions become slower as  $S$  gets smaller, and plans do sometimes fail to converge. In the analysis that follows, I require population convergence within 2%.

A general view of the quality of the maps is shown in the lower panel of Figure 2, with power diagram maps for Pennsylvania for each value of  $S$ . By shrinking counties towards their centroids, the borders of the simulated districts naturally align with county lines. Examples from other states and algorithms are included in the Supplementary Materials.

This Section uses these maps to address three questions:

1. Does the scaling method improve the average rate  $\bar{\mathcal{L}}$  at which county co-residents are assigned to the same legislator? Smaller values of the scaling  $S$  imply more-shrunken counties or “cheaper” perimeters between counties. If the spatial approach is a success, lower values of  $S$  should be associated with higher  $\bar{\mathcal{L}}$ .
2. How do automated methods contrast with enacted maps, for  $\bar{\mathcal{L}}$ ?
3. Is the partisan composition of states’ congressional delegations affected by the choice of algorithm or by the value scaling parameter  $S$ ? If the partisan composition varied predictably with  $S$ , politicians on the left or the right would rightly object that algorithmic map-making obfuscated partisan preference without actually removing it. There is therefore a “preference” for a flat partisan performance, as a function of  $S$ .

**Figure 2:** Illustration of the scaling method and results of the automated procedures, for Pennsylvania. Upper panel illustrates the scaling method. Counties' population centroids are marked by black circles. Counties are shrunk towards these points. The nominal boundaries are in black, and the scaled geometries are shaded. The lower panel presents representative results from the power diagram method. County lines are again shown in black, while district lines are shown in red. Moving from left to right, the scaling becomes more extreme, moving from  $S = 0.9$  to 0.7 and 0.5 of the nominal length scale. The red district lines are better-aligned with the black county lines for extreme scaling (0.5) than modest scaling (0.9).

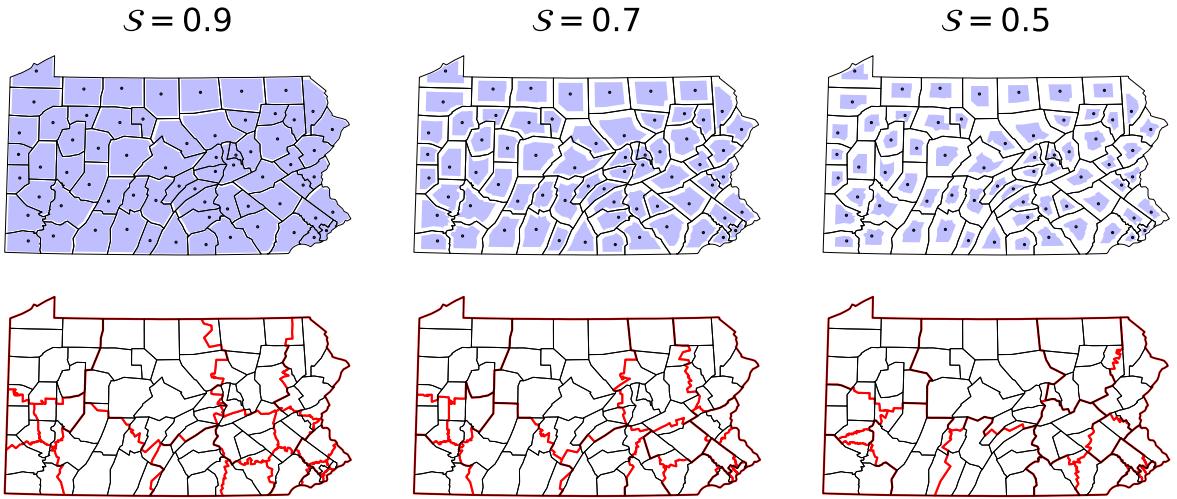
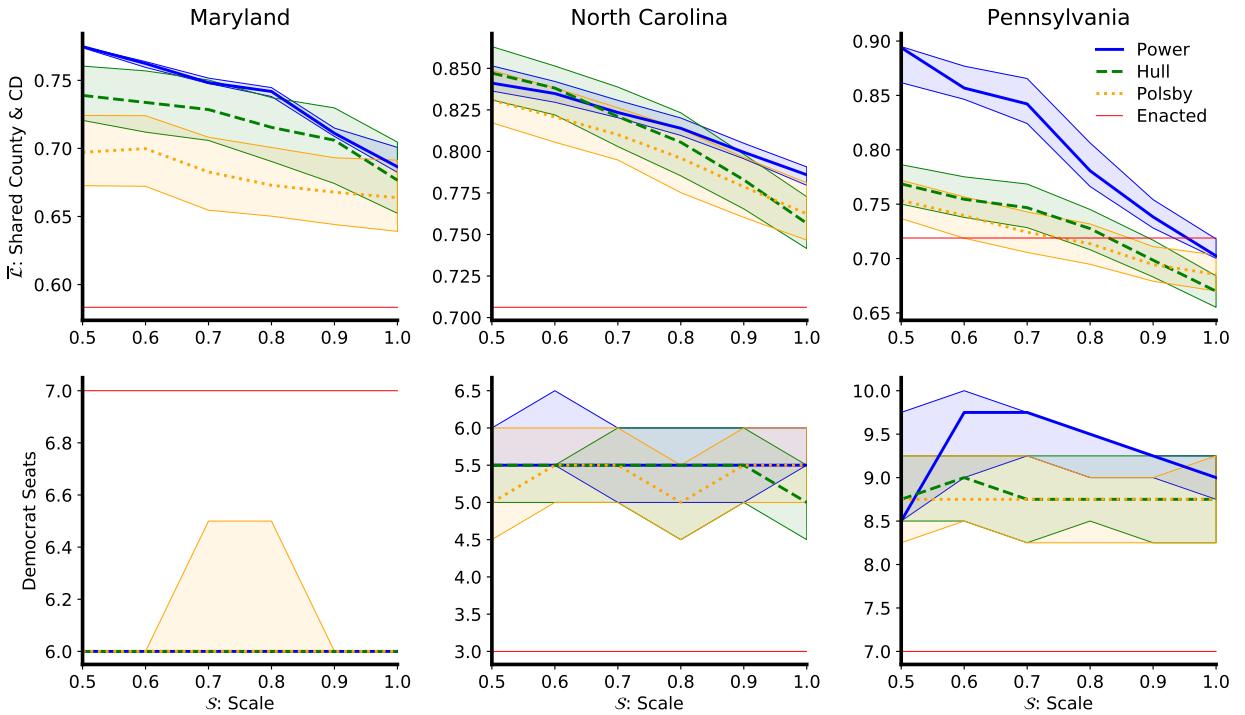


Figure 3 displays the partisan composition of the state congressional delegations, and the rates at which county co-residents share legislators, as a function of the scaling parameter  $S$  and for the three states studied. Medians and interquartile ranges are shown for each of the three regionalization methods (power diagrams, convex hull ratio, and IPQ). These results may be contrasted with the results according to the maps drawn by state legislatures after the 2010 Census.

The algorithms perform as designed. Smaller values of the scaling parameter reduce the level of county splits. In North Carolina and Maryland, the first quartile of  $\bar{L}$  for any of the automated methods exceeds that of the enacted map, for any value of  $S$ . In Pennsylvania,  $S$  may be set such that the median value of  $\bar{L}$  exceeds that of the enacted map, for any of the three algorithms. For power-digram derived maps, this threshold is reached at  $S \leq 0.9$ ; it is for  $S \leq 0.8$  for the convex hull method and  $S \leq 0.7$  for IPQ. The straightforward spatial manipulations provide a sliding scale between spatial compactness and community co-assignment. Based on  $\bar{L}$  as defined, the automated methods easily outperform the enacted maps in reducing divisions of existing political subdivisions. This is important, because it adds to the range of objectives that automated methods can be used to satisfy.

The lower panels of Figure 3 show partisan outcomes as a function of  $S$ . In Maryland and North Carolina, the partisan composition of the delegation is unresponsive to the value of  $S$ . This is also true in Pennsylvania for the convex hull and IPQ methods, but among the power diagram maps, the median number of Democratic representatives does vary by 1 seat over the domain shown. (This means one more Democrat *and* one less Republican.) There can be little doubt that, forced to use power diagrams, Pennsylvania Republicans would advocate a scaling of  $S = 1.0$  rather than 0.7, allowing them to expect 9 seats instead of just 8.25. But such a choice would not yield reliable gains in other states. Moreover, for any value of  $S$  – even for power diagrams of Pennsylvania – the partisan composition of the simulated delegations is far from that realized under the partisan plans. In all three states, the party controlling the districting process enacted maps to that

**Figure 3:** Impact on county co-assignment and partisan seat share, as a function of scaling. Upper panel shows community co-assignment and lower panel shows partisan outcomes, in Maryland, North Carolina, and Pennsylvania. Thick lines show the median plan while the shading covers the interquartile range. The horizontal axis ranges from  $S = 0.5$  to  $1.0$  (no scaling). Values for the statewide plans as enacted for the 114th Congress are also shown. The scaling parameter has a clear impact on the degree to which communities (counties) are divided, but has minimal effect on partisan outcomes. For reference, Maryland was apportioned 8 seats after the 2010 Census, North Carolina 13, and Pennsylvania 18.



party's advantage – fewer or more Democratic seats than expected under the non-partisan algorithms. Those advantages are large with respect to the variation of non-partisan plans. In none of the states can  $S$  be set to match the partisan performance of the enacted map.

## 5. Conclusions

In *Rucho v. Common Cause*, the Supreme Court ended decades of speculation, deciding not to provide relief for partisan gerrymandering. Reformers must now look towards other avenues. History suggests that Congressional action is possible. From the early days of computing, politicians and justices expected that the problem would be solvable with computers. The algorithms and computational power are finally catching up to the task.

This project has aimed to expand the possible objectives that can be implemented and assessed. This aim of assigning jurisdictions and communities to single legislative districts is a universally recognized, “traditional districting principle.” This paper has shown that through straightforward scaling of the geographic inputs, algorithms designed to optimize districts’ spatial “compactness” can be repurposed to reduce the divisions of existing communities. Statewide plans derived with various degrees of scaling are broadly consistent in terms of their partisan outcomes. In recently litigated states of Maryland, North Carolina, and Pennsylvania, the simulated partisan outcomes differ markedly from enacted ones.

There is much yet to do. This project has focussed on political subdivisions with clear existing ge-

ographies, but the problem is much more nebulous as we turn toward “communities of interest” that lack uncontested definitions. I have not evaluated the impact of these measures on minority representation. Although the partisan outcomes are fairly unresponsive to  $S$  within each method, the methods are not equally immune to manipulation. This analysis is situated in the context of a Federal reform requiring automated districting. Other questions – for instance, how dramatically a state’s partisanship could be manipulated while preserving compactness or respecting communities – would require different analysis.

The legislative districting process at its best elevates communities through maps; it is an expression of democratic values. Today’s gerrymandered maps instead evidence a corrupt and broken system. Open debate about political values is needed to settle what principles should be enshrined in law, and how they might be implemented rigorously and without bias. This debate will require careful mathematical and algorithmic work to formalize legitimate districting principles and understand the practical impacts of methods. This project has shown how existing algorithms can be extended to capture one more of these principles – the traditional concern of respecting existing political communities.

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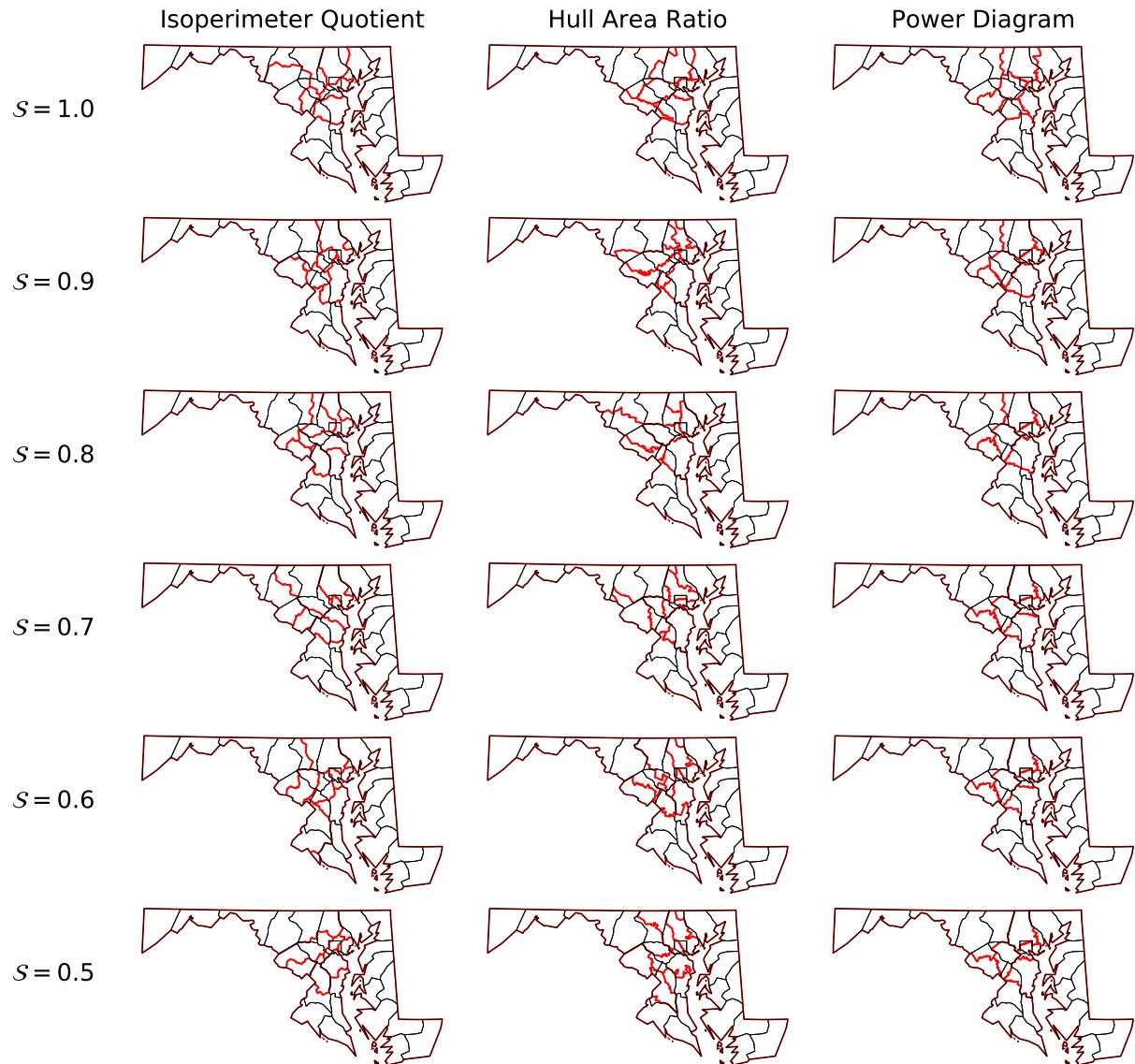
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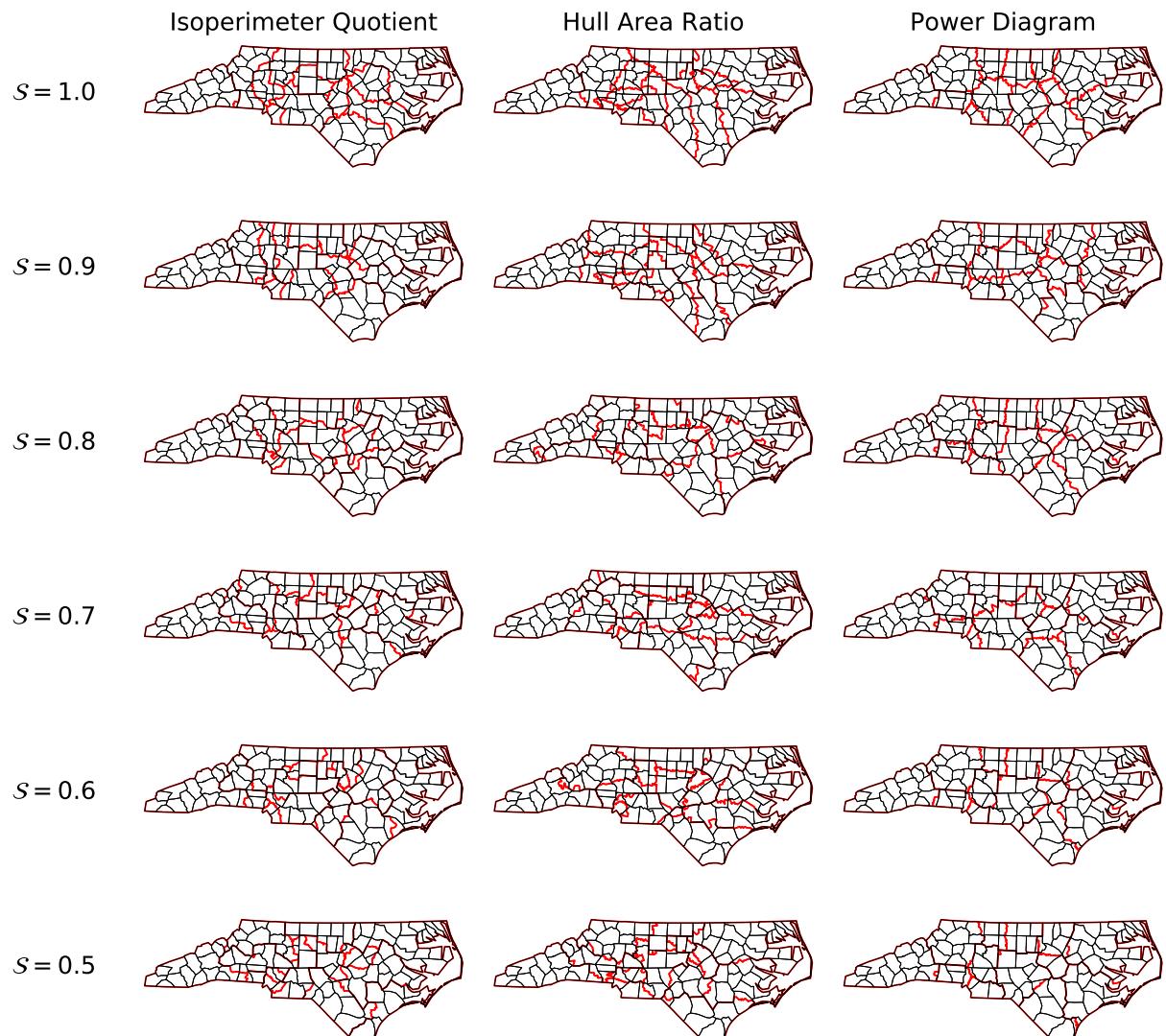
## Supplementary Material: Results for all States and Methods

A randomly selected collection of maps is presented in Figures [SM.1](#), [SM.2](#), and [SM.3](#), for the Maryland, North Carolina, and Pennsylvania, at each spatial scale considered the main text.

**Figure SM.1:** Representative (randomly selected) districts generated for Maryland, for each method and value of  $S$ .



**Figure SM.2:** Representative (randomly selected) districts generated for North Carolina, for each method and value of  $S$ .



**Figure SM.3:** Representative (randomly selected) districts generated for Pennsylvania, for each method and value of  $S$ .

