

REVIVING LEGISLATIVE AVENUES FOR GERRYMANDERING REFORM WITH A FLEXIBLE, AUTOMATED TOOL

JAMES SAXON

Harris School of Public Policy and Center for Spatial Data Science, University of Chicago

ABSTRACT. After seeking a “manageable standard” to apply to claims of partisan gerrymandering for over three decades, the Supreme Court has finally given up the chase, ruling that such claims are nonjusticiable. What is to be done? An extended history of successful Congressional action suggests that the legislative pathway is more practical than often believed. Statutory requirements also make it possible to consider a broader suite of districting objectives.

This paper presents flexible new software, and a framework for evaluating the practical implications of explicit objectives. I apply this approach to the conditions last required by Congress, generating equipopulous, contiguous, and compact districts. Among these conditions, the formal definition of *compactness* has proven contentious. Does it matter? I contrast the representation of the political parties and of racial and ethnic minorities, under plans optimized according to eighteen different definitions of compactness. On these grounds, the definitions are markedly consistent. These methods may be extended to alternative districting objectives and criteria.

E-mail address: `jsaxon@uchicago.edu`.

Date: August 5, 2019.

I would like to thank Dan Black, Luc Anselin, Julia Koschinsky, Scott Ashworth, Tom Coleman, Gary King, Jonathan Rodden, Marc Farinella, Marc Elias, Ben Ginsberg, Harry Hirsch, the journal’s editor Jeff Gill, and four anonymous reviewers for valuable feedback and comments. Supplementary materials for this article are available on the Political Analysis website and replication materials are retrievable from the Political Analysis Dataverse ([Saxon 2019](#)).

1. INTRODUCTION

Partisan gerrymandering, the manipulation of legislative districts for political gain, distorts democratic representation and undermines confidence in government. The United States will next redistrict after the 24th Census in 2020-2022, and gerrymandering stands as a fulcrum in the balance of power between the parties. Most of the recent literature has framed this issue through the courts, with authors jockeying to assist the Justices with a “manageable standard” for identifying gerrymanders. (Grofman and King 2007; Stephanopoulos and McGhee 2015) But after three decades seeking such a standard, the Supreme Court in *Rucho v. Common Cause* gave up the chase, ruling that partisan gerrymandering is a nonjusticiable, political question. Chief Justice Roberts concluded his opinion by noting that the legislative “avenue for reform established by the Framers, and used by Congress in the past, remains open.” (588 U.S., 2019, 33) This paper focusses on that Federal, legislative approach.

The Constitution gives Congress power over the “Times, Places and Manners of holding [its] Elections.” Historically, Congress used that power to regulate the form of constituencies in the House. The Apportionment Act of 1842 required that representatives be elected from contiguous, single-member districts (Cong. Globe 1842); subsequent Acts stipulated that districts be equipopulous (17 Stat. 28 1872) and compact (31 Stat. 733 1901). Those requirements were reiterated in 1911 (37 Stat. 13) but lost in 1929. (Cong. Rec. 1929) Though both houses revived them in 1967, the bill failed in conference. (Cong. Quarterly 1968) In the past half-century, Congress has neglected its responsibility for fair districts, but the issue was reintroduced with the first House bill of the 116th Congress. (For a legal history, see Appendices G and H.)

This paper evaluates the potential impact of reviving statutory regulation of the form of Congressional districts. It provides credible tools for comparing districting objectives. I begin

from the requirements as last enacted by Congress: equipopulation, contiguity, and compactness. Doing so, I immediately confront an old conceptual hurdle: what does it mean for a district to be “compact”? Equipopulation and contiguity have undisputed mathematical definitions, but there are dozens of formal alternatives for compactness. These definitions quantify the closeness of people within a district, the length of its perimeters, or the similarity of its shape to a circle (see Section 2 or [Angel et al. \(2010\)](#); [Chambers and Miller \(2010\)](#); [Ehrensburg \(1892\)](#); [Forrest \(1964\)](#); [Fryer and Holden \(2011\)](#); [Grofman \(1985\)](#); [Hofeller and Grofman \(1990\)](#); [Niemi et al. \(1990\)](#); [Polsby and Popper \(1991\)](#); [Reock \(1961\)](#); [Schwartzberg \(1966\)](#); [Young \(1988\)](#)). Which definition ought to be applied? Does it matter? Does a choice among algorithms or objectives amount to a political choice between parties?

I adopt a simplified – but constitutionally and historically justified – framing, in which Congress assumes wholesale control of the process. I focus on explicit procedures for strict maximization of objectives. This is how the courts have enforced equipopulation: exactly, not approximately. This simplifies the districting process; it treats the forms of congressional districts in the same mechanical terms as the apportionment of seats to states. I implement these procedures and objectives in flexible automated districting software, that is a core contribution of this paper. This software makes it possible to generate statewide plans for different districting objectives.

Automated districting is in a class of problems not exactly solvable with computers. The number of potential solutions is combinatoric in the number of inputs, so an exhaustive search for the unique, best solution is not possible. Computer scientists call this type of problem “NP-hard.” To make progress, heuristic, iterative strategies may be employed. Searches from random initializations (seeds) terminate at local, not global, extrema. Repeated initializations and searches allow for the assembly of a collection of high-quality maps for each version of compactness. This makes it possible to quantitatively evaluate the impact on representation

of different explicit objectives and algorithms. The software created for this project is distinguished from earlier work by the diversity of objective functions implemented, and by its ability to generate a larger number of plans for larger states than past projects. It builds on a long history of “automated districting,” stretching from [Weaver and Hess \(1963\)](#) at the dawn of computation, to modern projects like [Altman and McDonald \(2011\)](#) who wrote software but did not exercise it, and [Fryer and Holden \(2011\)](#) who generated individual maps using “power diagrams.” More recently, [Chen and Rodden \(2013; 2015\)](#) and [Cho and Liu \(2016\)](#) deployed automated districting procedures to generate “populations” of districts, and measure if enacted plans were outliers with respect to these distributions. That work has shown that enacted plans are indeed outliers with respect to specific automatically generated distributions. But are all distributions of maps consistent?

For each map generated by the software, I evaluate districts’ practical characteristics by aggregating voting returns and constituent populations. This completes the translation of definitions of compactness into collections of maps and thence into distributions of practical outcomes. I construct these distributions in ten states, based on available data. I demonstrate that the choice among compactness objectives and algorithms is not in itself a loaded, implicitly political issue. Even though the shapes of generated districts change between the definitions, the various methods treat the two parties consistently in terms of seat shares, vote shares, and the number of competitive races. I show how this consistency arises and how the “compact” maps compare to the status quo. The work thus departs from past reviews (like [Niemi et al. 1990](#)) by suggesting that – in this context – compactness is well-defined in practice. New legislation can and should make a compactness requirement procedurally rigorous. This legal clarity need not divide the parties.

Since I am investigating a federal, legislative reform that would supersede existing law, the Voting Rights Act (VRA) does not apply. Still, minority representation matters and

it would be affected by reform. Per the intent of Section 2 of the VRA, I evaluate the impact of a compactness requirement on minority representation. I use a single, representative definition of compactness to district the entire United States and find the net change in minority representation to be small. I briefly discuss alternative ways of framing this issue.

The methods presented are applicable and the conclusions on compactness pertinent, beyond the frame of a Federal, legislative requirement. A majority of state constitutions require compact legislative districts, and such requirements have been the basis for successful litigation at the state level, as in *League of Women Voters v. the Commonwealth of Pennsylvania*. A number of states are also considering constitutional amendments to create independent commissions for redistricting. The methods of this paper facilitate data-driven debate over the meaning and implications of compactness, as both legal standards and as objectives for commissioners. At the Federal level, establishing a claim of vote dilution in violation of Section 2 of the Voting Rights Act requires litigants to demonstrate that a minority is “geographically compact” (the first of the *Gingles* preconditions). The methods here offer a determination of that compactness that is more realistic and sophisticated than existing approaches. More broadly, the courts have consistently enumerated compactness as a “traditional districting principle” but have struggled to enforce it, since its formal meaning has been ill-specified.

It is also hoped that this software will help foster a more formal and quantitative debate over aspatial objectives – both in isolation and in tandem with the spatial ones presented here. Despite its wide use, some critics have argued that compactness is a poor (or incomplete) criterion. (Grofman 1985) The automated software developed can be extended with formal objectives for other “traditional districting principles.” For example, the aim of respecting existing political subdivisions can be integrated as an objective in the software or used post hoc to evaluate districts (Appendix E). Maps with better prospects for minority candidates can be selected from an existing distribution (Appendix F).

Congress has enormous latitude over the forms of congressional districts. It was granted this power intentionally and explicitly by the founders, with the responsibility of ensuring uniform and unbiased regulation of representation. (Kurland, *et al.* 2000) It has of late evaded this responsibility. This paper aims to revive interest in this power and responsibility, by evaluating the impact of Congress picking up where it left off in 1967 and requiring compact districts. Districting via compactness would shift the partisan composition of the delegations of gerrymandered states like Maryland and North Carolina; these shifts are consistent across every formal definition of compactness considered in this project. On its own, compact districts would also carry a slight reduction in minority representatives.

Seen broadly, this paper offers a flexible tool and quantitative framework for evaluating trade-offs between formal districting objectives in legislative and academic debate.

2. COMPACTNESS

Compactness is a succinct proxy for proximity-based communities. It reflects the American norm of geographic representation for the House that, though not mandated by the Constitution, was clearly expected by the founders and is today required by statute. This Section presents a number of common expressions for measuring compactness, which will be used as objective functions in the optimization procedure presented in Section 3.1. It describes some of their limitations, in that context. It also presents several stand-alone procedures or algorithms for generating compact districts.

2.1. Defining Compactness. In an early review of compactness, Niemi *et al.* (1990) remarked that it is “multidimensional.” As illustrated in Figure 1, the single word corresponds to many different notions. While a circle is broadly understood to be compact, shapes may be non-compact by being disperse (or distended), highly indented, or dissected. A single shape may be judged more or less compact based on the proximity of the elements it contains.

Each separate notion corresponds to a separate mathematical expression. Not all notions of compactness are created equal. In this work, I reject mathematical definitions that are not invariant under rotations or scaling. The compactness of a district does not change when measured in miles instead of kilometers or when surveyed with a compass instead of a sextant. This precludes concepts like the total perimeter of a district, or the ratio of its North-South and East-West extents. I further require that each measure of compactness be normalized to 1, and that a larger number is always more compact. This requirement makes compactness comparable across states and districts with very different population density, and makes it easier to combine it with other constraints, namely equipopulation.

Most measures of compactness are composed of ratios of lengths, areas, or populations of the district, with respect to a reference shape derived from the district. Figure 2 illustrates the common reference shapes. Four circles are defined: the largest inscribed circle (LIC), the smallest circumscribing circle (SCC), and the circles of equal area and equal perimeter. I define the radius of the equal area circle $R \equiv \sqrt{A/\pi}$, and call the circle of radius R centered at the district's centroid C_R . The convex hull (CH) is defined as the smallest convex polygon that encloses a district; it is the shape a rubber band would make if wrapped around it.

For each shape, I denote its surface (or shape) by S and its area by A , the perimeter by P and the length of that perimeter by ℓ . The population contained within the shape is p . One may additionally define radii to the centers of population ρ_p or area ρ_A , and distances to the perimeter d_P or between two points d_{ij} . The shortest internal path δ_{ij} is the distance between two points, constrained to lie *within* the shape (see Figure 2). In what follows, I denote intersections by $\cap$ and averages by $\langle x \rangle$. I use subscripts to denote variants of these quantities, for the derived shapes.

With these definitions in hand, it is straightforward to define “classical” compactness measures from various ratios. The measures are tabulated in Table 1 and described below.

2.1.1. *Isoperimeter quotient.* Perhaps the most-famous measure of compactness is the isoperimeter quotient (IPQ); it is defined as the ratio of a shape’s area to that of a circle of equal perimeter. A circle of circumference ℓ has area $\pi(\ell/2\pi)^2 = \ell^2/4\pi$, so the IPQ simplifies to $4\pi A/\ell^2$. In the districting literature, it is often attributed to [Polsby and Popper \(1991\)](#), while $\text{IPQ}^{-1/2}$ is associated with [Schwartzberg \(1966\)](#). The IPQ is mainly sensitive to the perimeter; it responds little to broad deformations of the shape. It and other perimeter measures exhibit subtle definitional issues, because geographic boundaries like coastlines often have “fractal” properties in the sense that their lengths depend on the scale at which they are measured. Nevertheless, the method is computationally simple, readily understood, widely used, and fairly performant. I drop a number of monotonic transformations of the IPQ (like the Schwartzberg measure).

2.1.2. *Convex hull ratios.* The convex hull may be used to define numerous metrics by dividing the areas or populations in the district with those in the hull. To recognize the existing geometry of the state (whose borders may not be compact), the population or area of the hull may be limited to citizens or land within the same state. I implement the area ratio A/A_{CH} and “population polygon” method $p/p_{CH, \text{state}}$. These typically privilege convex shapes and result in maps with clean, convex districts that may however be fairly disperse or “long.”

2.1.3. *Moments of Inertia.* The moment of inertia I is a dispersion measure defined by the weighted distances squared of the elements of a district to a fixed point. [Weaver and Hess](#) implemented its application to the districting problem as early as [1963](#). In this case, the weights w_i of the cells are their populations or areas, and the fixed point considered is the center of mass (either area or population). For discrete elements i on the surface S , this is
$$I = \sum_{i \in S} w_i \rho_i^2.$$

A uniform circular disk of equal area is typically used as the reference shape. The moment of inertia of such a disk with respect to its center is $A \sum_i w_i / 2\pi$. For areal weights this simplifies to $A^2 / 2\pi$ and for a disk of population N it is $NA / 2\pi$. Since large moments of inertia denote less compactness, the normalization is in the numerator: $2\pi (\sum_i w_i \rho_i^2) / (A \sum_i w_i)$.

2.1.4. *Inscribed and Circumscribing Circles.* In 1892, [Ehrenburg](#) proposed considering the ratio of a shape’s area to that of its LIC or SCC: A_{LIC}/A and A/A_{SCC} . [Reock](#) proposed the latter measure again in [1961](#), and it sometimes bears his name in the districting literature. In practice I have found that they require heavy-handed optimization with ad hoc ‘fixes’ (see Appendix [C.4.1](#)). The problem is that any change to the shape that does not touch the circle is equivalent, so that there is no penalty for ‘tentacles’ and no ‘smooth path’ towards a global minimum.

2.1.5. *Exchange index.* [Angel et al. \(2010\)](#) propose to calculate “exchange” as the ratio of the areas of the intersection of an equal-area circle centered at the district’s centroid, with the district itself: $A(S \cap C_R)/A$. The larger the fractional intersection, the more compact the shape. Because it privileges modifications to districts’ boundaries that place more of the area close to the center, the definition has a smooth “path” towards an optimal configuration and works well in automated settings. It is in some sense a dispersion measure.

2.1.6. *Mean, Dynamic, or Harmonic Radius.* The mean radius is the average value of the radius ρ to the district centroid. The dynamic and harmonic radii instead express ρ^2 and $1/\rho$, respectively. ([Frolov 1975](#)) All three are areal dispersion measures. Integrating the radius ρ over a surface S of area A , the mean radius is thus $(\int_S \rho dS)/S$, the dynamical radius is $\sqrt{(\int_S \rho^2 dS)/A}$, and the harmonic radius is $A/\int_S dS/\rho$. Each of these may be normalized by the corresponding value for a circle of equal area: $2R/3$, $R/\sqrt{2}$, or $R/2$, respectively. Since radii larger than a circle’s are *less* compact, the normalizations go in the numerator.

2.1.7. *Distance to Perimeter.* The average distance from a point in a shape to its perimeter d_P is compared to a circle, for which the value is $R/3$. More-compact districts have less of their area close to the perimeter.

2.1.8. *Path Fraction.* [Chambers and Miller \(2010\)](#) propose a measure of “bizarreness,” which reduces to the probability that the shortest in-state path between two people in the district, is itself contained within the district. The intuition is that a representative should not have to leave her district when driving from one voter to another. For people i and j , this is $\left(\sum_i \sum_j \lfloor d_{ij}/\delta_{ij} \rfloor\right) / N^2$.

2.1.9. *Interpersonal Distance/Power Diagrams.* [Fryer and Holden \(2011\)](#) use the total distance squared between people. If the people are aggregated into cells (here, census tracts) i with populations w_i and separated from neighbors j by distance d_{ij} , this is $\sum_i \sum_j w_i w_j d_{ij}^2$. Fryer and Holden demonstrate that partitions that optimize this measure are additively-weighted power diagrams (like a Voronoi diagram). Taking compact to mean “proximate,” they prove that these partitions are “optimally compact.” In [Section 2.2.1](#), I describe an explicit algorithm for power diagrams, but a normalized measure may also be defined by dividing the average interpersonal distance by the corresponding value for a circle of equal area, $128R/45\pi$.¹

2.1.10. *Axis Ratio.* The simple width to length ratio W/L is not very sensitive as a compactness measure: depending on the precise definition, a spindly “X” may be as compact as a square. But it is in fact used (at least, by Iowa) and it is readily calculable. The width and length may themselves be defined in a number of ways; I calculate W/L as the ratio of the

¹This normalization can be derived by using the law of cosines to calculate the distance from a point with $\theta = 0$ to another arbitrary point with $\theta \in \{0, \pi\}$, and integrating, taking care to weight the radius to get a uniform distribution on the disk: $\left(\int_0^\pi \int_0^1 \int_0^1 r_1 r_2 \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta} dr_1 dr_2 d\theta\right) R / \left(\int_0^\pi \int_0^1 \int_0^1 r_1 r_2 dr_1 dr_2 d\theta\right) = 128R/45\pi$.

eigenvalues of the two principal components of the population point cloud in the projected geometry.

2.1.11. *Visual Test.* The visual test (Young 1988) – sometimes jokingly called the interocular (it hits you between the eyes – Grofman 1991) or obscenity test (“I know it when I see it,” per Potter Stewart) – is in fact a serious legal and diagnostic tool. Justice O’Connor wrote in *Shaw v. Reno* that “reapportionment is one area in which appearances do matter” (509 U.S. 630, 1993). Recently, Chou et al. (2012, 2014) and Kaufman et al. (2017) have elicited visual feedback on district plans from both experts and laypeople, to understand what people perceive as unfair.

Moreover, the visual test is an indispensable diagnostic for debugging and evaluating if the code for other metrics are “working.” In that sense, it represents my own inescapable bias for this project: it is the threshold where maps looked sensible.

It is worth acknowledging that there do exist definitions of compactness that are not included above. There *is* a limit to what is computationally feasible for automation. For example, Angel et al. (2010) define a “traversal index” by dividing the average length of internal paths between points on a district’s perimeter by the corresponding value for a circle. Computationally, this is simply too demanding. Evaluating this method would entail reevaluating the shortest paths for every potential move, which is computationally unfeasible.

Part of the aim of this research is to motivate a broad strategy for comparing formal of districting objectives. This project could well be extended by implementing additional spatial or aspatial objectives. This is illustrated briefly in Appendix E, for communities of interest.

2.2. **Procedures.** In addition to metrics – scalars that can be used as objective functions in an arbitrary optimization – a number of algorithms or procedures have been defined for generating a compact districting for a state.

2.2.1. *Power Diagrams.* The first of these algorithms is the power-diagram method discussed above (2.1.9). My implementation is similar to Fryer and Holden’s, and proceeds as follows:

- (1) Regions r are defined by a center $\mathbf{x}_r$ and power λ_r . The initial centers are chosen randomly and the initial powers are set to 0.
- (2) Cells c located at $\mathbf{x}_c$ are assigned to region r by $\operatorname{argmin}_r (|\mathbf{x}_r - \mathbf{x}_c|^2 - \lambda_r^2)$.
- (3) The region centers $\mathbf{x}_r$ move slowly towards the region centroids and the powers λ_r increase or decrease so as to equalize the regions’ populations.

Steps 2 and 3 repeat until a convergence threshold is reached.

2.2.2. *Split-Line Algorithm.* The split-line algorithm iteratively splits the state’s regions. Regions (districts) r of population p_r containing $s_r > 1$ seats, are split in two pieces of population $p_r \lfloor s_r/2 \rfloor / s_r$ and $p_r \lceil s_r/2 \rceil / s_r$, along the shortest possible line. This proceeds until each region has a single seat. The algorithm was first conceived by Forrest (1964), and a slightly different approach is laid out by Spann et al. (2007)

2.2.3. *Areal or Population Radii.* For comparison purposes, I have included a distance-based assignment approach, similar to Chen, Rodden, and Cottrell (2013; 2015; 2016). In short, I trade cells between districts to minimize the cell’s squared distance to the population or area centroid. My method differs from theirs in that I weight the squared distances by the radius squared of the equal-area circle (in other words, the area divided by π). This makes the algorithm scale-invariant. I also allow the algorithm to run long beyond population convergence, to obtain more-compact districts. This approach can be naturally subsumed in the “general objective function” approach used for the other compactness scores.

This section has presented various meanings of compactness, and reviewed existing mathematical definitions of the term. I now turn to deploying these definitions, to algorithmically generate compact districts.

3. AUTOMATED DISTRICTING: THE C4 SOFTWARE

After each federal census, the states are apportioned representation in the House of Representatives proportional with their populations. The states are then tasked with assigning these seats to equipopulous, single-member districts of contiguous area.

This Section presents this problem mathematically and describes the software implemented to address it computationally. This project differs from earlier catalogs of definitions and past efforts at automation in the diversity of objectives implemented. I am therefore able to quantitatively assess the practical impact of alternative definitions of compactness. [Appendix B](#) reviews past work on this problem.

3.1. The Constraint Problem. Formally, electoral districting is a graph partitioning problem. The task is to partition a set (state) into N non-overlapping, contiguous, equipopulous regions r (congressional districts), while optimizing the compactness of those regions. I do this using discrete cells c of population p^c (census tracts). Each cell is a node in the graph of the state, and the nodes are connected by edges if they are contiguous (share perimeter). The graph of the state itself must be connected: for any two nodes on the graph, there must exist a path between them. (See [Appendix C.2](#) for details on islands and enclaves.)

The regions are then connected subgraphs of the state, and partition it: every cell in the state must belong to exactly one region. I denote the set of cells (nodes) in each region by X^r . The regions' populations are the sum of their cells' populations, $p^r = \sum_{c \in X^r} p^c$. The target population across regions p_{target} is equal to the population of the state, divided by N . The compactness of a region is a function of its cells, $\mathcal{C}(X^r)$. I will refer to the region that contains c by $r(c)$.

The contiguity requirement is algorithmically enforced: no change that results in a disconnected graph for any region is ever considered. To formalize this, it is useful to define the set of

nodes $X^{r'}$ that are not in X^r but are adjacent to a node in it, and are not themselves cut-nodes of their current region (their removal would not break its connectedness). Considering a cell c in $X^{r'}$ and X^s , I then define the union of X^r with one additional node c by $X_{+c}^r \equiv X^r \cup \{c\}$, and the set with one node removed by $X_{-c}^s \equiv X^s \setminus \{c\}$.

The contiguity requirement is thus built in to the procedure. The equipopulation constraint and compactness objective are explicitly optimized using a greedy search that proceeds cyclically over the N regions.

Naïvely, one might define a combined objective function, incorporating the compactness and population count of each region. In each iteration a region r would annex the cell $c \in X^{r'}$ whose reassignment from its current region resulted in the largest improvement in the combined objective function of the two regions. Along these lines, the population objective for each region might then take the form $\mathcal{P}(p_r) = -(|p_r/p_{\text{target}} - 1|/\Delta)^\alpha$, with Δ an allowable tolerance from p_{target} , and α a tunable parameter that I set to 4. The gradient of the population constraint would thus plummet as p^r/p_{target} approached within Δ of 1 (since the parenthesis is less than 1, raised to the fourth), but it would dominate the spatial part when $|p_r/p_{\text{target}} - 1| > \Delta$. One would then consider changes in this objective from moving a cell c from region r to s : $\mathcal{P}(p^r - p^c) + \mathcal{P}(p^s + p^c)$. This approach fails because the cells do not have equal population. Restricted to discrete trades, far from equilibrium, cells with larger population will always move first. Roughly speaking, the step size is much longer among more-populous cells but may not lead in the direction of steepest descent.

A small modification of the above suffices, but comes at the price of an explicit objective function. I define the population difference function by

$$\mathcal{P}(p^r, p^s) \equiv \text{sign}(p^r - p^s) \left(|p^r/p_{\text{target}}^r - p^s/p_{\text{target}}^s|/\Delta \right)^\alpha. \quad (1)$$

This expression depends only on regions, and is independent of cells. The population constraint thus impacts the choice of region to trade with, while the choice of cell *along* that border is left to the compactness scores $\mathcal{C}$. As above, this term dominates the compactness measure when two regions' population difference exceeds Δp_{target} but is very small when equipopulation is satisfied.

Each iteration on a region r culminates by its annexing the cell that maximizes the combined population and compactness function:

$$\arg \max_{c \in X^{r'}} \left[\mathcal{P}(p^r, p^{r(c)}) + \mathcal{C}(X_{+c}^r) - \mathcal{C}(X^r) + \mathcal{C}(X_{-c}^{r(c)}) - \mathcal{C}(X^{r(c)}) \right]. \quad (2)$$

The optimization procedure begins by seeding the N regions with N random cells, and the regions initially grow by subsuming unassigned cells u . Since the unassigned area has target $p_{\text{target}}^{r(u)} = 0$, the population score to transfer out of it is infinite. In practice, I replace this score with a large number so that the $\arg \max$ is well-defined, but the behavior is unaltered: the regions quickly converge to cover the state. The procedure thus partitions the state while respecting the contiguity of regions, and optimizing for equipopulation and compactness.

I also implement a modification of this procedure. In addition to one-directional moves, it is efficient to be able to trade cells between regions. If this functionality is activated, one trade is allowed per cycle, for which $c \in X^{r'}$ and $c' \in (X^{r(c)'} \cap X^r)$ yield the largest gain in compactness:

$$\arg \max_{c, c'} \left[\mathcal{C}(X_{+c, -c'}^r) - \mathcal{C}(X^r) + \mathcal{C}(X_{-c, +c'}^{r(c)}) - \mathcal{C}(X^{r(c)}) \right]. \quad (3)$$

3.2. Computational Implementation. A core contribution of this project is the software used to generate optimized districting plans. The software is called *C4*, for “contiguity-constrained clustering in c++.” *C4* is open-sourced and freely available on [GitHub](#). Key features are presented in greater detail in [Appendix C](#).

To begin, a user loads the cells for a state along with their adjacency matrix (shared perimeters). To be able to enforce contiguity, the statewide plan must initially be connected; islands' connections to the mainland may be specified explicitly, but *C4* also has a module to handle this automatically. *C4* also subsumes regions that are connected to the main graph by a single cut vertex, since it is definitionally impossible to reassign the cut vertex without breaking contiguity.

The search then begins with a random draw without replacement of one cell for each region. The hill-climbing procedure detailed above then begins. Critical to the algorithm's performance is its enforcement of region contiguity using integer programming. This is done by requiring that any cell removed from a region leave its neighbors in a single, connected subgraph. The search terminates after a configurable number of cycles with no improvement.

The software includes several of the standard metaheuristic strategies. Tabu lists ([Glover 1989](#)) are in fact used for some compactness objectives, with individual cells precluded from moving for a fixed number of iterations after reassignment. I further implement two non-standard procedures. First is a “de-strand-ing” method that removes strands of cells that cannot be removed by the cell-by-cell search. Second is a method for re-starting the search, by splitting in two the region with the worst compactness score and merging two other regions. Still, users should note that the software makes no guarantees in regards the globally optimality of solutions; such is the nature of NP-hard problems.

4. SPATIAL, DEMOGRAPHIC, AND ELECTORAL DATA

This Section describes the required geographic, demographic, and election data. Geographic and demographic data are drawn from the US Census Bureau. Electoral data are far less standardized. I rely on both past efforts to assemble precinct-level returns, as well as some

data directly from the states. The processed data and replication code for this project can be found at the Political Analysis Dataverse. ([Saxon 2019](#))

4.1. Spatial and Demographic Data. The fundamental cells for map generation are 2015 census tracts. The geometries used are the Census’s Cartographic Boundary shapefiles, and tract populations are from the 2015 American Community Survey. ([US Cenus Bureau 2016, 2018](#)) I have generated topologies from each state geometry using PostGIS 2.1. To reduce perimeter measures’ sensitivity to highly-indented (fractal) perimeters as along waterways, I have simplified the edges of the topology. The simplification is nominally to the 10km level; however if this would result in a new intersection (node/face/edge), the simplification threshold is successively halved until no intersection would result. For optimization and analysis, I project each state into its local EPSG a coordinate reference system – usually a transverse Mercator or Lambert conformal conic, but sometimes Albers equal area projection. The datum is NAD83(HARN) and the units are meters. In states with multiple local projections, I select the center-most one. The list is derived from [spatialreference.org](#), and is included [with the replication materials](#).

For comparisons with historical Congressional Districts, I have used the 107th, 111th, and 114th Congresses, which were drawn after the last three censuses. ([US Cenus Bureau 2012a, 2013a, 2015](#))

4.2. Election Returns. I employ precinct-level returns for Presidential elections mapped by [Ansolabehere and Rodden \(2011a,b\)](#) for Florida (2008) and Illinois (2008). For Maryland (2008), Pennsylvania (2000-2012), and Texas (2000-2008) I have merged election returns by [Ansolabehere et al. \(2015\)](#) with Voter Tabulation Districts (VTDs) from the Census (2010) and Texas (2016). For Pennsylvania in 2012, the precinct names were slightly inconsistent; manual corrections and (human-verified) “fuzzy” matches were necessary. I supplement these with

data directly from the states for Illinois (2016), Louisiana (2012, 2016), Maryland (2016), Minnesota (2008-2016), North Carolina (2012, 2016), Tennessee (2016), Texas (2012-2018), and Wisconsin (2004-2016). ([Illinois State Board of Elections 2017](#); [Louisiana House of Representatives 2012, 2016](#); [Louisiana Secretary of State 2012, 2016](#); [Minnesota Geographic Information Services 2009, 2016, 2017](#); [North Carolina State Board of Elections 2012, 2013, 2016a,b](#); [Tennessee Comptroller of the Treasury 2012](#); [Tennessee Secretary of State 2016](#); [Texas Legislative Council 2008, 2014, 2016, 2017, 2018](#); [Wisconsin Legislative Technology Services Bureau 2017, 2018](#)) For Maryland in 2016, the polling places and not precincts were available; I therefore use the former. The Illinois precincts have changed significantly since the 2010 Census release, and I have updated the precincts for Cook, Dupage, and Lake Counties. ([Ferruzzi 2016](#); [Levy 2016](#); [DuPage County GIS 2016](#); [Lake County, Illinois 2016](#)) Together, these cover most of the changes and more than half of the state’s population. The rest of the state is matched by precinct and county name. In Louisiana, North Carolina, and Tennessee, where early, absentee, and provisional voting are recorded at the county level, I divide these votes among precincts in proportions equal to the polling-place share of the county vote for each party.

5. THE POLITICAL CONSISTENCY OF COMPACTNESS DEFINITIONS

This Section contrasts the spatial and political characteristics of maps generated with the *C4* software with those from enacted plans. For this work, the fundamental cell size is the census tract, and equipopulation is required at the 2% level (with a few exceptions, below). Readers may object that federal law allows districting at the census block level and that the Supreme Court has rejected any *de minimis* threshold of equipopulation. These choices have the obvious advantage of speeding up the computation, though most of the algorithms work fine at the block group level. But they should also be considered in the context of the legislative

approach that motivates this paper. Census tracts are designed to encapsulate relatively-homogeneous populations and their use can be thought of as a minimal regard for “preservation of communities.” When Congress last considered legislation to require equipopulous districts in 1967, it was at the 10% level – a looser threshold in better balance with other districting objectives. ([Cong. Quarterly 1968](#)) As to the Court’s enforcement of its “one-person one-vote” doctrine, it would be faced with Congress’s explicit Article 1, Section 5 authority over the elections and qualifications of its members.

I have generated a thousand maps per measure for each state for which I have voting data, using distributed computing. The exceptions are the path fraction, where I have generated only 280 maps per state, and the split-line algorithm, which is deterministic and yields a unique solution per state. The optimization procedures do sometimes fail to converge within population tolerance; the following analyses are therefore restricted to those maps with population deviations less than 2%. The population convergence issue is more acute with the axis ratio method and for Texas, and in these cases I allow a 5% deviation; for axis ratio maps *of* Texas, a 10% threshold is allowed. The split-line algorithm generates a 2.1% deviation in Illinois and that solution is retained.

One must be precise about the statistical nature of this collection of maps. The algorithms are initialized by selecting one cell (census tract) to seed each district, without replacement. This is a bona fide random draw. Each states’ seeds are generated fifty times; each seed is “restarted” 20 times, resulting in different solutions. The optimized districts are of course not random.

After some general observations about the visual consistency of methods, I analyze the consistency of the compactness measures in two ways. First, I study the political outcomes of the populations of “optimized” maps: the seat share and competitiveness. I then turn briefly to the potential impacts for minority representation.

5.1. Observations on Optimized Maps. In Figure 3, I present a representative collection of maps drawn from a single seed. Additional plans can be explored interactively, [online](#). Differences between methods emerge as expected. Axis ratio is simply ineffective. Isoperimeter quotient contains “somewhat lumpy” shapes with smooth perimeters. The hull-based measures, along with the power diagram and split-line algorithms, produce convex shapes with straight lines. It is interesting to consider the non-trivial relationships between the many methods. Power diagrams imply convex shapes that would result in good scores for hull population or hull area, but the converse is not necessarily true: convex shapes can be very distended (disperse) while power diagrams usually are not. A convex shape will contain all of the paths between people in the district and will therefore have a “Path Fraction” of 1 (§2.1.8), and a shape with a perfect “Path Fraction” likewise implies a high convex hull population ratio (§2.1.2), but the paths through phase space towards these optima are not generally the same.

Across measures, the varying treatment of Pittsburgh is particularly notable: some algorithms divide it in many pieces (distance to the areal center or split-line) while others cut a circle around the city (exchange, harmonic radius, or inscribed circles). It is this variation in the treatment of urban (in America, Democratic) voters, that raises the possibility of bias from compactness. Is choosing an objective equivalent to choosing a winner?

5.2. The Political Consistency of Optimized Maps. The seat share and competitiveness of simulated districts are derived by reaggregating precinct-level voting data from presidential elections, described in Section 4. Presidential elections are used to avoid uncontested races and reduce incumbency effects. The procedure depends on consistency between presidential and congressional races. It is also an approximation in the sense that local candidates could better tack to individual constituencies, and even change their strategies as a function of the district lines. To mitigate this concern, multiple elections are presented when available, to give a sense of geographically-realistic distributions with different statewide vote shares.

Each individual map results in a certain number of projected wins for Republicans and a complementary number for Democrats; each measure’s population of maps thus corresponds to a distribution of seats for each election. The same procedure is followed for the actual enacted maps from the last three districting cycles. In this way, the internal consistency of the simulated maps may be evaluated, and as a group contrasted with the enacted maps. Results are shown for four elections in Pennsylvania in Table 2. The other nine states – Florida, Illinois, Louisiana, Maryland, Minnesota, North Carolina, Tennessee, Texas, and Wisconsin – are available in Appendix A. In Table 3, I tabulate the expected number of “competitive” seats with margins of victory less than 5% and plot the distribution of vote shares for each state and method.

Three major themes stand out from these results. The first theme is the remarkable consistency of the seat shares among the eighteen algorithms and metrics shown, and across substantial variation in the statewide two-party vote shares. This pattern is reproduced for all of the states studied.² This result suggests that from the perspective of the seat share, the choice of the definition of compactness is immaterial in this automated context. A similar but weaker result emerges from the competitiveness of the seats in Table 3. Though the agreement is not quite as tight as for the seat shares, the various metrics put fairly consistent numbers of seats in play.

The second observation is that although Democrats won each of the four elections shown in Table 2 by at least 2.5%, they capture a majority of the eighteen seats only in the 2008 election, which Barack Obama won by more than 10%. This thus reproduces the earlier results on “unintentional gerrymandering” by Chen and Rodden: Pennsylvania Republicans enjoyed a structural advantage from their demography, independent of any machinations by the State Legislature. This is due to Democrats’ “inefficient” clustering in Philadelphia and Pittsburgh.

²The consistency is again reproduced when using races for the US Senate in Texas. See Appendix D.

The same effect is also visible in the vote shares of Table 3, in particular for Illinois. Chicago voters are overwhelmingly liberal, and Democratic candidates can expect margins of victory that are “inefficient” for the party as a whole. This is apparent in the heavy left-hand tails. As in Pennsylvania, the effect is accentuated by the fact that the major metropolis is in the corner of the state: it is hard to dole these voters out to swing districts. Maryland has a different story. Democrats again have a substantial majority, receiving around 60% of the vote in presidential elections. Naïvely, this might be sufficient for the entire state to “go blue.” But Maryland Republicans are protected by their geography: they are concentrated in the panhandle and Eastern Shore which, for any reasonably compact partitioning, are sliced off as two safe Republican districts. The take-away is the unsurprising fact that each state’s political geography affects the representation for the two parties.

Still, returning to Pennsylvania and comparing the expectations from the simulated maps to that of the map enacted for the 114th Congress, it is apparent that the Pennsylvania Republicans enjoy an additional one to two seat advantage through their control of the districting process. This advantage persists over several elections, and in 2000 and 2012 the expectation of 6 seats for Democrats is completely outside the distribution of seats simulated using any compactness method. That map was struck down before the 116th Congress. Similar pictures emerge in Maryland and North Carolina, where the Democratic and Republican majorities enacted plans that yield seat shares outside the distribution from simulations. The last observation is thus that the simulation provides a baseline “unbiased” level, from which the observed deviations on enacted maps evidence intentional gerrymandering. Crucially, this conclusion is extremely robust to the method employed to generate the counterfactual.

5.3. Impacts on Minority Representation. Before advocating automated, objective-based approaches for districting, it is necessary to understand and consider the potential impacts on minority representation. Minority voting rights are constitutionally and statutorily protected

under the 15th Amendment and the Voting Rights Act (VRA). Though the VRA is somewhat cumbersome, it has been effective: minorities in the US are represented at rates far closer to proportionality than in peers like Germany, France, and the United Kingdom that lack explicit legal frameworks for ensuring their representation.³ (Donovan 2007; Stephanopoulos 2013; U.S. House of Representatives, Office of the Historian 2017a,b) However, the Supreme Court in 2013 dramatically curbed the preventative force of the VRA, with *Shelby County v. Holder*. That decision struck down the “coverage formula” that determined which jurisdictions were required to seek “preclearance” before changing their elections laws. Without the coverage formula, preclearance lies dormant ahead of the 2022 redistricting. Asserting Federal authority over the forms of Congressional districts would supersede existing law; this offers new alternatives for guaranteeing minority representation (for US Congressional districts, only). What would be the impact of compactness for minorities?

To study this, I have generated compact districts for the entire country using the power diagram algorithm. As noted, power diagrams are closely related to optimizing on the interpersonal distance. A Principle Component Analysis of the compactness of historical Congressional districts from the last three districting rounds, shows that it is correlated to the first component of the PCA at 95% (Appendix I). They are also extremely fast to generate.

Armed with a population of maps, I aggregate the ethnic and racial composition of census tracts to calculate the Black and Hispanic fraction of each simulated district as I had previously done for the precinct-level votes. In Figure 4, I present the number of seats (actual and simulated) where the Black or Hispanic share of the voting age population (VAP) exceeds a given threshold. This exercise is grounded on the premise that the fraction of a district’s VAP belonging to a racial or ethnic majority is the key determinant in its electing a minority representative. The vertical distance between the dashed and solid lines gives a flavor for

³Among developed nations, New Zealand’s dedicated seats for Maōri have been more successful. Canada also has fairly high rates of minority representation in the lower house of its parliament. (Chowdry 2015)

the change in minority representation from moving from the status quo to power diagram based districting, if a single threshold triggered minority representation. The lines intersect in both panels. This suggests that if a high minority share were required to elect a minority representative, the currently enacted plans would result in higher minority representation than the power diagram maps. Conversely, if a low minority share were required, the power diagram maps could result in higher minority representation. Unlike the party share, where 50% is clearly the relevant threshold, there is no axiomatic level of minority presence for a minority candidate to be elected.

Acknowledging the complexity involved in measuring such a threshold, and recognizing that using a single value country-wide is a gross simplification, I offer two simple approaches. The first is to identify the value of the VAP fraction f , such that the number of constituencies with minority share greater than or equal to f is matched by the number of minority representatives actually elected in the 115th Congress. At this level, each district with a larger minority share that *does not* elect a minority representative is compensated by another district with a lower share that *does* elect one. This is illustrated by a thin horizontal line in Figure 4. There were 40 Hispanic and 46 Black representatives in the 115th Congress,⁴ which translates into fractions of 30% for Blacks and 41% for Hispanics.

Alternatively, Cameron, Epstein, and O'Halloran (1996; 1999) famously evaluated a probit model with the minority share of the VAP as the independent variable and an indicator for a minority representative as the dependent variable. Taking the simplest possible model with minority share x , minority representation r , intercept α , and slope β , I fit the normal ogive $r = \Phi(\beta x + \alpha)$. This simple approach ignores the interplay between Black and Hispanic

⁴I consider the union of entries from the [U.S. House of Representatives, Office of the Historian \(2017a,b\)](#) and the [U.S. House of Representatives, Press Gallery \(2017\)](#). The delegates from the District of Columbia, the Marianna Islands, Puerto Rico, and the US Virgin Islands are all minorities, but I do not include them in this count. In California's 34th district, Jimmy Gomez replaced Xavier Becerra in a special election. They are both Hispanic, and I count the district once.

populations and the role of primaries in determining the (Democratic) candidate ([Lublin 1999](#); [Lublin et al. 2009](#)), and it is also markedly coarser than the local, Ecological Inference approach usually adopted in litigation. But the intent here is also very different: to evaluate the impact of a national change for which voting data are unavailable. In the 115th Congress, the 50% crossing point for this model is 35% for Blacks and 52% for Hispanics.

Intuitively, these thresholds correspond to a sizable majority of the majority party. The thresholds are higher for Hispanic districts, due to higher eligibility and turnout among Blacks than Hispanics. Using the first approach to the threshold, the enacted and compactness-based maps produce an almost equal number of minority seats in the region of interest. To interpret the probit models, one must take the sum over districts of the probabilities of electing a minority. Doing this for the actual districts yields 46.0 Black and 40.0 Hispanic representatives, compared to the true values of 46 and 40. The same sum of probabilities with the simulated maps yields 42.1 Black and 37.0 Hispanic representatives. According to the probit, a pure power-diagram approach would then lead to a 8% reduction in Black representation and a 7% reduction in Hispanic representation.

In practice however, VRA compliance means that real districts constructed with a high minority share also typically have a partisan composition favorable to minority candidates. The power diagram generation does not fine-tune this correlation, and power diagram districts may therefore require a higher raw minority VAP fraction to elect a minority candidate. This caveat implies that the estimates of minority representation under power diagram districts are likely inflated.

This said, in the past several Congresses, growth in minority representation has outpaced growth in the minority share of the population. This suggests that the “threshold” for minority representatives is falling. This point is reinforced by earlier work by [Grofman et al. \(2001\)](#). To the left of the intersections between curves of the enacted and simulated maps in [Figure 4](#),

the simulated maps yield higher minority fractions. If the threshold continues to fall, the minority representation under a compactness-based approach may exceed that from the current patchwork of judgment-based law.

Further, it is worth noting that the machinery already described provides the means for sidestepping the potential reductions in minority representation apparent in this analysis. One could include minority representation in the objective function, or preferentially select maps with better minority prospects from the sets of automatically-generated compactness-based districts. The latter approach is demonstrated in Appendix F.

6. CONCLUSIONS AND FUTURE WORK

This paper has presented *C4*, a credible automated districting software that implements many compactness definitions and districting algorithms from the previous literature. This software facilitates quantitative, outcome-based discussion of districting objectives. Past compendia of compactness measures have not systematically implemented the proposed definitions in automated procedures, and have therefore not been able to contrast the implications of the proposals.

Using this software, I have generated populations of contiguous and equipopulous maps for a number of states, optimized for each compactness measure and algorithm. Aggregating votes from presidential elections into the simulated districts, I have projected the “winner” of each district in each election. I have thus transformed the populations of maps into distributions of vote shares for seats, and seat shares for states. The party vote shares across seats and elections reflect the political geography of the states; their distributions are remarkably consistent across methods, for each state. In particular, there is good agreement in the integrals of vote shares above and below 0.5 (seat shares for the two parties), as well as between 0.475 and 0.525 (number of competitive seats). This consistency between methods suggests that the

“unintentional gerrymandering” effect established by Chen and Rodden is quite robust to the specific, geometric definition of compactness. Using power diagrams to simulate hundreds of maps for every state in the country, I find that a purely compactness-based approach would result in small but noticeable reductions in minority representation.

This work offers a new strategy for evaluating the impacts of formal objectives for legislative districts, in the context of Congressional action on gerrymandering in the United States. It could be extended by incorporating alternative initialization strategies including graph theoretic approaches and hierarchical partitioning, or adding algorithms and compactness definitions. Studies like those of [Chou et al. \(2014\)](#) and [Kaufman et al. \(2017\)](#), that elicit human feedback on which measures yield the most appealing solutions could also be informative. Figure 3 shows that different objectives generate different shapes, and one can imagine ranking objective measures according to their subjective performance. Given the interest in protecting communities of interest and political subdivisions, it is worth formalizing and implementing objective functions to encode these adjacencies, and other “normative” goals (minority representation, competitive districts, etc.). Such measures could then supplement or replace the spatial terms in the objective function, as illustrated in Appendix E.

Since the balance of this document has suggested that the various definitions of compactness are similar in their effects on representation, it is natural to ask which one to use. Power diagrams are a strong candidate. They converge quickly and reliably, and result in clean, convex polygons – which is generally desirable but sometimes results in split cities (see Figure 3). [Fryer and Holden \(2011\)](#) showed that power diagrams minimize the average interpersonal distance of co-constituents in the state. It is a non-trivial benefit that this distance is easy to comprehend. The interpersonal distance is also a good proxy for the other compactness measures. A principal component analysis of the compactness measures of historic districts yields a first component that is correlated at $\rho = 0.95$ with the interpersonal distance (Appendix I).

On the other hand, power diagrams are implemented as a stand-alone algorithm, and do not integrate as well with other objectives.

After selecting a measure, how compact should the districts in a map be? This paper has considered the behavior at convergence – maximal compactness – with no latitude left to the states. Critics might protest that this strategy imperils other traditional principles, like respect for political subdivisions or communities of interest. But that is not so: those objectives can be formalized and included in the optimization (Appendix E). As described in the context of minority representation (and demonstrated in Appendix F), one could also select plans from the distribution that satisfy some other objective. The algorithms presented here maximize compactness locally and not globally, so one might simply choose the single most-compact map. Strict maximization has the strong appeal of transparency. It is bundled with the immense but worthwhile challenge of formalizing and forging consent over objectives for democratic representation. If a looser standard were imposed by Congress, the appropriate analysis of impacts would shift to how effectively the standard constrained partisan cartographers. The present work has treated statewide compactness as the average (or sum) over districts, but more-nuanced criteria could be defined. The software developed could be extended and applied to each of these analyses.

Outright maximization also offers important preventative effect. The “bright line” of the Supreme Court’s “one person, one vote” standard (*Gray v. Sanders* 372 U.S. 368, 1963) virtually eliminated malapportioned districts. The Justices long sought a similarly “precise rationale” for adjudicating partisan gerrymandering, and have finally called off the search. Congress has the power to provide that rationale, as it did to protect minority voting rights. But the VRA’s history also highlights the challenges of nuance. The “Senate Factors” used to identify discriminatory election laws have forced the justices to exercise their gut judgement over the “totality of circumstances,” case by case. This conceptual obscurity has done little to

dampen legislators’ appetites; the successes of the VRA have been achieved only through (or despite) relentless litigation. North Carolina’s twelfth district has reached the Supreme Court on seven occasions since the 1992 redistricting: *Shaw v. Reno* (1993), *Shaw v. Hunt* (1996), *Hunt v. Cromartie* (1999), *Easley v. Cromartie* (2001), *Cooper v. Harris* (2017), and *Rucho v. Common Cause* (in both 2018 and 2019). With the VRA weakened by the *Shelby County* decision, preventative measures are needed to ensure minority voting rights. Strict, centralized maximization would deliver uniform districts more efficiently than nuanced criteria.

Of course, a new Apportionment Act is hardly the only proposed solution to political gerrymandering, nor is it the only one hinging on a clearer definition of compactness. As already noted, states can implement compactness requirements, and these may provide the footing for successful legal challenges. When the Pennsylvania Supreme Court determined in 2018 that a districting plan violated the state’s constitution, it struck down the plan and the US Supreme Court denied a petition to stay that ruling. In most of the developed world, redistricting is performed by independent commissions. ([Stephanopoulos 2013](#)) These commissions must be charged with their objectives; should compactness rank among them, a single definition would provide clarity and consistency to the process. The present work has suggested that a choice among definitions need not be politics in disguise.

The Supreme Court’s long search for a “clear and manageable standard” for adjudicating partisan gerrymanders has come to an end with *Rucho v. Common Cause*. Reformers must now look elsewhere. It is has been an explicit aim of this paper to direct attention at the Federal level towards the constitutionally-sanctioned, legislative pathways. Congress has a history of exercising this power. To revive and enrich debate over districting objectives, this paper has offered credible software and methods for evaluating and contrasting their practical impacts. Automated generation of compact districts is not the only solution to gerrymandering, and it is perhaps not a complete one. I contend, however, that optimization

of explicit objectives is likely to be a useful tool for any solution – legislative or otherwise. A quantitative understanding of the implications of formal districting objectives is critical to both research and reform. I look forward to the continued refinement of algorithms and explicit objectives, for compactness and other districting aims.

REFERENCES

- An Act for the Apportionment of Representatives to Congress among the several States according to the ninth Census, 42d Congress, 2d session; 17 Stat. 28 (1872). [🔗](#)
- An Act Making an apportionment of Representatives in Congress among the several States under the Twelfth Census, 56th Congress, 2d session; 31 Stat. 733 (1901). [🔗](#)
- An Act For the apportionment of Representatives in Congress among the several States under the Thirteenth Census, 62d Congress, 1st session; 37 Stat. 13 (1911). [🔗](#)
- Altman, M. and McDonald, M. 2011. “BARD: Better Automated Redistricting.” *Journal of Statistical Software* **42** (1): 1-28 (2011). DOI: [10.18637/jss.v042.i04](https://doi.org/10.18637/jss.v042.i04)
- Angel, S., Parent, J., and Civco, D. L. 2010. “Ten compactness properties of circles: measuring shape in geography.” *Canadian Geographer* **54** (4): 441-461 (2010). DOI: [10.1111/j.1541-0064.2009.00304.x](https://doi.org/10.1111/j.1541-0064.2009.00304.x)
- Ansolabehere, S. and Rodden, J., “Florida Data Files.” *Harvard Dataverse*. 2011a. DOI: [1902.1/16797](https://doi.org/10.1902.1/16797)
- Ansolabehere, S. and Rodden, J., “Illinois Data Files.” *Harvard Dataverse*. 2011b. DOI: [1902.1/15845](https://doi.org/10.1902.1/15845)
- Ansolabehere, S., Palmer, M., and Lee, A., “Precinct-Level Election Data, 2002-2012.” *Harvard Dataverse*. 2015. DOI: [1902.1/21919](https://doi.org/10.1902.1/21919)
- Cameron, C., Epstein, D., and O’Halloran, S. 1996. “Do Majority-Minority Districts Maximize Substantive Black Representation in Congress?” *The American Political Science Review* **90**

- (4): 794-812 (1996). DOI: [10.2307/2945843](https://doi.org/10.2307/2945843)
- Chambers, C. P. and Miller, A. D. 2010. "A Measure of Bizarreness." *Quarterly Journal of Political Science* **5** (1): 27-44 (2010). DOI: [10.1561/100.00009022](https://doi.org/10.1561/100.00009022)
- Chen, J. and Rodden, J. 2013. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures." *Quarterly Journal of Political Science* **8** (3): 239-269 (2013). DOI: [10.1561/100.00012033](https://doi.org/10.1561/100.00012033)
- Chen, J. and Rodden, J. 2015. "Cutting Through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders." *Election Law Journal* **14** (4): 331-345 (December, 2015). [🔗](#)
- Chen, J. and Cottrell, D. 2016. "Evaluating partisan gains from Congressional gerrymandering: Using computer simulations to estimate the effect of gerrymandering in the U.S. House." *Electoral Studies* **44** 329-340 (2016). DOI: [10.1016/j.electstud.2016.06.014](https://doi.org/10.1016/j.electstud.2016.06.014)
- Cho, W. K. T. and Liu, Y. Y. 2016. "Toward a Talismanic Redistricting Tool: A Computational Method for Identifying Extreme Redistricting Plans." *Election Law Journal* **15** (4): 351-366 (December, 2016). DOI: [10.1089/elj.2016.0384](https://doi.org/10.1089/elj.2016.0384)
- Chou, C., Kimbrough, S., Sullivan-Fedock, J., Woodard, C. J., and Murphy, F. H. 2012. "Using Interactive Evolutionary Computation (IEC) with Validated Surrogate Fitness Functions for Redistricting," at *14th Annual Conference on Genetic and Evolutionary Computation*, Philadelphia, Pennsylvania, USA, 1071–1078 DOI: [10.1145/2330163.2330312](https://doi.org/10.1145/2330163.2330312)
- Chou, C., Kimbrough, S. O., Murphy, F. H., Sullivan-Fedock, J., and Woodard, C. J. 2014. "On Empirical Validation of Compactness Measures for Electoral Redistricting and Its Significance for Application of Models in the Social Sciences." *Social Science Computer Review* **32** (4): 534-543 (2014). DOI: [10.1177/0894439313484262](https://doi.org/10.1177/0894439313484262)
- Chowdry, A. 2015. "Record number of visible minority MPs elected to Commons." *The Globe and Mail* (October 20, 2015). [🔗](#)

- Cong. Globe, 27th Cong., 2d sess. Vol 11, pp. 407 and 788-790 (agreement on the principle), 449-451 and 786-788 (doom), 790 (state strategies). (May 2, 1842; June 4 and 10, 1842) [↗](#)
- Cong. Rec., 70th Cong., 2d sess. Vol 70, pp. 1496, 1499, 1584, 1602, 1604. (January 10-11, 1929) [↗](#) The statute as enacted is [46 Stat. 26](#).
- Donovan, B. 2007. "Minority Representation in Germany." *German Politics* **16** (4): 455-480 (2007). DOI: [10.1080/09644000701652482](#)
- Ehrenburg, K. 1892. "Studien zur Messung der horizontalen Gliederung von Erdräumen. Mit 2 Tafeln.." *Verhandlungen der Physikalisch-medicinischen Gesellschaft zu Würzburg* **25** 29-72 (1892). [↗](#)
- Epstein, D. and O'Halloran, S. 1999. "A Social Science Approach to Race, Redistricting, and Representation." *The American Political Science Review* **93** (1): 187-191 (1999). DOI: [10.2307/2585770](#)
- Ferruzzi, A., "Historical - ccgisdata - Election Precinct Data - 2015 to 2016." *Cook County Government, Open Data*. Updated March 1, 2016. Accessed May 1, 2017. [↗](#)
- Forrest, E. 1964. "Apportionment by Computer." *American Behavioral Scientist* **8** (4): 23 (1964). DOI: [10.1177/000276426400800407](#)
- Frolov, Y. S. 1975. "Measuring the Shape of Geographical Phenomena: A History of the Issue." *Soviet Geography* **16** (10): 676-687 (1975). DOI: [10.1080/00385417.1975.10640104](#)
- Frolov cites K. Ehrenburg's "Studies on the measurement of the horizontal shapes of areas," in the *Verhandlungen der Physikalisch-medincinischen gesellschaft zu Würzburg* (1892).
- Fryer, R. G., Jr.. and Holden, R. 2011. "Measuring the Compactness of Political Districting Plans." *The Journal of Law and Economics* **54** (3): 493-535 (2011). DOI: [10.1086/661511](#)
- Glover, F. 1989. "Tabu SearchPart I." *ORSA Journal on Computing* **1** (3): 190-206 (1989). DOI: [10.1287/ijoc.1.3.190](#)
- Gray v. Sanders, 372 U.S. 368 (1963). [↗](#)

- Grofman, B. 1985. "Criteria for Districting: A Social Science Perspective." *UCLA Law Review* **33** 77-184 (1985). [🔗](#)
- Grofman, B. 1991. "Lessons from the American Experience: What Happens After One Person-One Vote? Implications of the United States Experience for Canada." In *Drawing Boundaries: Legislatures, Courts, and Electoral Values*. Saskatoon, Saskatchewan: Fifth House Publishers.
- Grofman, B., Handley, L., and Lublin, D. 2001. "Drawing Effective Miority Districts: A Conceptual Framework and Some Empirical Evidence." *North Carolina Law Review* **79** (2001). [🔗](#)
- Grofman, B. and King, G. 2007. "The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after LULAC v. Perry." *Election Law Journal: Rules, Politics, and Policy* **6** (1): 2-35 (February, 2007). DOI: [10.1089/elj.2006.6002](https://doi.org/10.1089/elj.2006.6002)
- Hofeller, T. and Grofman, B. 1990. "Comparing the Compactness of California Congressional Districts under Three Different Plans: 1980, 1982, and 1984.." In *Effective Representation*. New York: Agathon.[🔗](#)
- Kaufman, A., King, G., and Komisarchik, M. 2017. *How to Measure Legislative District Compactness If You Only Know it When You See It*. (Harvard University; Cambridge, MA). [🔗](#)
- Kurland, P. and Lerner, R. 2000. *The Founders' Constitution*, Vol. 2, Article 1, Section 4, Clause 1 University of Chicago Press. [🔗](#) For arguments on the use of Article 1, Section 4 for maintaining fair districts, see in particular Madison at the [Federal Convention](#) and back home at the [Virginia Ratifying Convention](#), or Appendix [G.1](#).
- Levy, J., "Precincts (current)." *Chicago Data Portal*. Updated August 23, 2016. Accessed May 1, 2017. [🔗](#)

- Lublin, D. 1999. "Racial Redistricting and African-American Representation: A Critique of 'Do Majority-Minority Districts Maximize Substantive Black Representation in Congress?'" *The American Political Science Review* **93** (1): 183-186 (1999). DOI: [10.2307/2585769](https://doi.org/10.2307/2585769)
- Lublin, D., Brunell, T. L., Grofman, B., and Handley, L. 2009. "Has the Voting Rights Act Outlived Its Usefulness? In a Word, 'No'." *Legislative Studies Quarterly* **34** (4): 525-553 (2009). DOI: [10.2307/20680256](https://doi.org/10.2307/20680256)
- Niemi, R. G., Grofman, B., Carlucci, C., and Hofeller, T. 1990. "Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering." *The Journal of Politics* **52** (4): 1155-1181 (1990). DOI: [10.2307/2131686](https://doi.org/10.2307/2131686)
- Polsby, D. D. and Popper, R. D. 1991. "The Third Criterion: Compactness as a Procedural Safeguard Against Partisan Gerrymandering." *Yale Law & Policy Review* **9** 301-353 (1991). [🔗](#)
- Reock, E. C. 1961. "A Note: Measuring Compactness as a Requirement of Legislative Apportionment." *Midwest Journal of Political Science* **5** (1): 70-74 (1961). DOI: [10.2307/2109043](https://doi.org/10.2307/2109043)
- Rucho v. Common Cause*, 588 U.S. ??? (2019). [🔗](#)
- Saxon, J., "Replication Data for: Reviving Legislative Avenues for Gerrymandering Reform with a Flexible, Automated Tool." *Harvard Dataverse*. 2019. DOI: [10.7910/DVN/NIPYJ8](https://doi.org/10.7910/DVN/NIPYJ8)
- Schwartzberg, J. E. 1966. "Reapportionment, Gerrymanders, and the Notion of Compactness." *Minnesota Law Review* **50** 443-452 (1966).
- Shaw v. Reno*, 509 U.S. 630 (1993). [🔗](#)
- Spann, A., Kane, D., and Gulotta, D. 2007. "Electoral Redistricting with Moment of Inertia and Diminishing Halves Models.." *UMAP Journal* **28** (3): 233 (2007). [🔗](#)
- Stephanopoulos, N. 2013. "Our Electoral Exceptionalism." *University of Chicago Law Review* **80** (769): (2013). [🔗](#)

Stephanopoulos, N. and McGhee, E. 2015. "Partisan Gerrymandering and the Efficiency Gap."

University of Chicago Law Review **82** (831): (2015). [🔗](#)

Weaver, J. B. and Hess, S. W. 1963. "A Procedure for Nonpartisan Districting: Development of

Computer Techniques." *The Yale Law Journal* **73** (2): 288-308 (1963). DOI: [10.2307/794769](https://doi.org/10.2307/794769)

Young, H. P. 1988. "Measuring the Compactness of Legislative Districts." *Legislative Studies*

Quarterly **13** (1): 105-115 (1988). [🔗](#)

Cong. Quarterly 1968. *Congress Fails to Adopt House District Standards*, Washington DC:

Congressional Quarterly, pp. 550–557. [🔗](#)

DuPage County GIS, "Election Precincts." Updated December 14, 2016. Accessed May 1,

2017. [🔗](#) Website points to current precincts.

Illinois State Board of Elections, "Election Vote Totals Results." Accessed May 7, 2017. [🔗](#)

Lake County, Illinois, "Voting Precincts." 2016. [🔗](#) Website points to current precincts.

Louisiana House of Representatives, "2012 Louisiana Precinct Shapefile." Updated February

14, 2013. Accessed May 16, 2017. [🔗](#)

Louisiana House of Representatives, "2016 Louisiana Precinct Shapefile." Updated September

9, 2016. Accessed May 16, 2017. [🔗](#)

Louisiana Secretary of State, "Election Results by Parish: Official Results for Election Date

11/06/2012 (Presidential Electors)." Accessed May 17, 2017. [🔗](#) Parish-level returns can be downloaded systematically, as shown in the replication repositories.

Louisiana Secretary of State, "Election Results by Parish: Official Results for Election Date

11/08/2016 (Presidential Electors)." Accessed May 17, 2017. [🔗](#) Parish-level returns can be downloaded systematically, as shown in the replication repositories.

Minnesota Geographic Information Services, "Election Results." *Minnesota Legislature*. Up-

dated February 12, 2009. Accessed November 17, 2017. <ftp://ftp.commissions.leg.state.mn.us/pub/gis/shape/elec08.zip>

Minnesota Geographic Information Services, “Election Results.” *Minnesota Legislature*. Updated November 9, 2016. Accessed November 17, 2017. <ftp://ftp.commissions.leg.state.mn.us/pub/gis/shape/elec2016.zip>

Minnesota Geographic Information Services, “Election Results.” *Minnesota Legislature*. Updated January 24, 2013. Accessed November 17, 2017. <ftp://ftp.commissions.leg.state.mn.us/pub/gis/shape/elec2012.zip>

North Carolina State Board of Elections, “SBE Precincts 09012012.” Updated August 31, 2012. Accessed May 7, 2017. [🔗](#)

North Carolina State Board of Elections, “Precinct Election Results: 11/06/2012.” Updated Jan 22 2013. Accessed May 6, 2017. [🔗](#)

North Carolina State Board of Elections, “Precinct Election Results: 11/08/2016.” Updated December 16, 2016. Accessed May 6, 2017. [🔗](#)

North Carolina State Board of Elections, “SBE Precincts 20161004.” Updated October 4, 2016. Accessed May 7, 2017. [🔗](#)

Tennessee Comptroller of the Treasury, “Map Selection.” Accessed August 14, 2017. [🔗](#) County files may be downloaded individually, as demonstrated in replication files. Modification dates by county range between June 7, 2012 and August 14, 2017 (in response to a correction, requested by the author).

Tennessee Secretary of State, “2016 Election: United States President, Results by Precinct.” Updated December 13, 2016. Accessed August 14, 2017. [🔗](#)

Texas Legislative Council, “2008 General Election Returns.” Accessed May 3, 2017. ftp://ftpgis1.tlc.state.tx.us/elections/2008_General_Election_Returns.csv

Texas Legislative Council, “2014 General Election Returns.” Accessed March 19, 2019. ftp://ftpgis1.tlc.state.tx.us/elections/2014_General_Election_Returns.csv

Texas Legislative Council, “2016 General Election Returns.” Accessed May 3, 2017. ftp://ftpgis1.tlc.state.tx.us/elections/2008_General_Election_Returns.csv

Texas Legislative Council, “Voting Tabulation Districts.” Updated February 7, 2017. Accessed May 3, 2017. ftp://ftpgis1.tlc.state.tx.us/2011_Redistricting_Data/VTDs/Geography/VTDs.zip

Texas Legislative Council, “2018 General Election Returns.” Accessed March 19, 2019. ftp://ftpgis1.tlc.state.tx.us/elections/2018_General_Election_Returns.csv

U.S. House of Representatives, Office of the Historian, “Black-American Representatives and Senators by Congress, 1870 - Present.” Accessed August 4, 2017a. [🔗](#)

U.S. House of Representatives, Office of the Historian, “Hispanic-American Representatives and Senators by Congress, 1822 - Present.” Accessed August 4, 2017b. [🔗](#)

U.S. House of Representatives, Press Gallery, “Demographics.” Accessed June 17, 2017. [🔗](#)

US Census Bureau, “Congressional District TIGER Shapefiles, 111th Congress.” Updated March 1, 2012. Accessed February 1, 2018. <ftp://ftp2.census.gov/geo/tiger/GENZ2010/> The Congressional District files are of the form gz_2010_[USPS]_500_11_500k.

US Census Bureau, “Congressional District TIGER Shapefiles, 107th Congress.” Updated June 20, 2013. Accessed February 1, 2018. [🔗](#)

US Census Bureau, “2015 TIGER/Line Shapefiles: Congressional Districts (114).” Updated August 13, 2015. Accessed February 1, 2018. [🔗](#)

US Census Bureau, “Cartographic Boundary Files - Shapefile.” Updated April 4, 2016. Accessed August 30, 2016. <ftp://ftp2.census.gov/geo/tiger/GENZ2015/shp/>

US Census Bureau, “American Community Survey 5-Year Data (2009-2017).” 2018. [🔗](#) Data series has been updated since data were retrieved.

Wisconsin Legislative Technology Services Bureau, “2002-2010 WI Election Data with 2011 Wards.” Updated October 3, 2017. Accessed May 15, 2017. [🔗](#) Note: dataset has been

updated and moved since data were retrieved.

Wisconsin Legislative Technology Services Bureau, “2012-2020 WI Election Data with 2017 Wards.” Updated October 19, 2018. Accessed May 15, 2017. [🔗](#) Note: dataset has been updated and moved since data were retrieved.

Section	Metric	Formula	Notable References
2.1.1	Isoperimeter Quotient (IPQ)	$4\pi A/\ell^2$	Polsby and Popper (1991)
2.1.2	Convex Hull Area Ratio	A/A_{CH}	
2.1.2	Population Polygon	$p/p_{CH,state}$	Hofeller and Grofman (1990)
2.1.3	Moment of Inertia	$2\pi \sum_i w_i \rho_i^2 / (A \sum_i w_i)$	Weaver and Hess (1963)
2.1.4	District to LIC Area Ratio	A_{LIC}/A	Ehrenburg (1892)
2.1.4	District to SCC Area Ratio	A/A_{SCC}	Reock (1961)
2.1.5	Exchange Index	$A(S \cap C_R)/A$	Angel et al. (2010)
2.1.6	Mean Radius ρ (to centroid)	$(2R/3) / ((1/A) \int_S \rho dA)$	Frolov (1975)
2.1.6	Dynamic Radius ρ^2 (to centroid)	$(R/\sqrt{2}) / \sqrt{(1/A) \int_S \rho^2 dA}$	Frolov (1975)
2.1.6	Harmonic Radius $1/\rho$ (to centroid)	$(R/2) / (A / \int_S dA/\rho)$	Frolov (1975)
2.1.7	Rohrbach Index	$(\int_S d_P dA) / (\pi R^3/3)$	Frolov (1975)
2.1.8	Path Fraction	$(\sum_{i \in S} \sum_{j \in S} \lfloor d_{ij}/\delta_{ij} \rfloor) / N^2$	Chambers and Miller (2010)
2.1.9	Minimum Distance (see also 2.2.1)	$\sum_{i \in S} \sum_{j \in S} w_i w_j d_{ij}^2$	Fryer and Holden (2011)
2.1.10	Width to Length	Eigenvalues	
2.1.11	Visual/Interocular Test	–	Young (1988), Grofman (1991)
2.2.1	Power Law (see also 2.1.9)	(Algorithm)	Fryer and Holden (2011)
2.2.2	Split-Line Algorithm	(Algorithm)	Forrest (1964)
2.2.3	Areal or Population Distance	(Algorithm)	Chen and Rodden (2013)

TABLE 1. Metrics of compactness, with formulae. See Section 2.1 for notation.

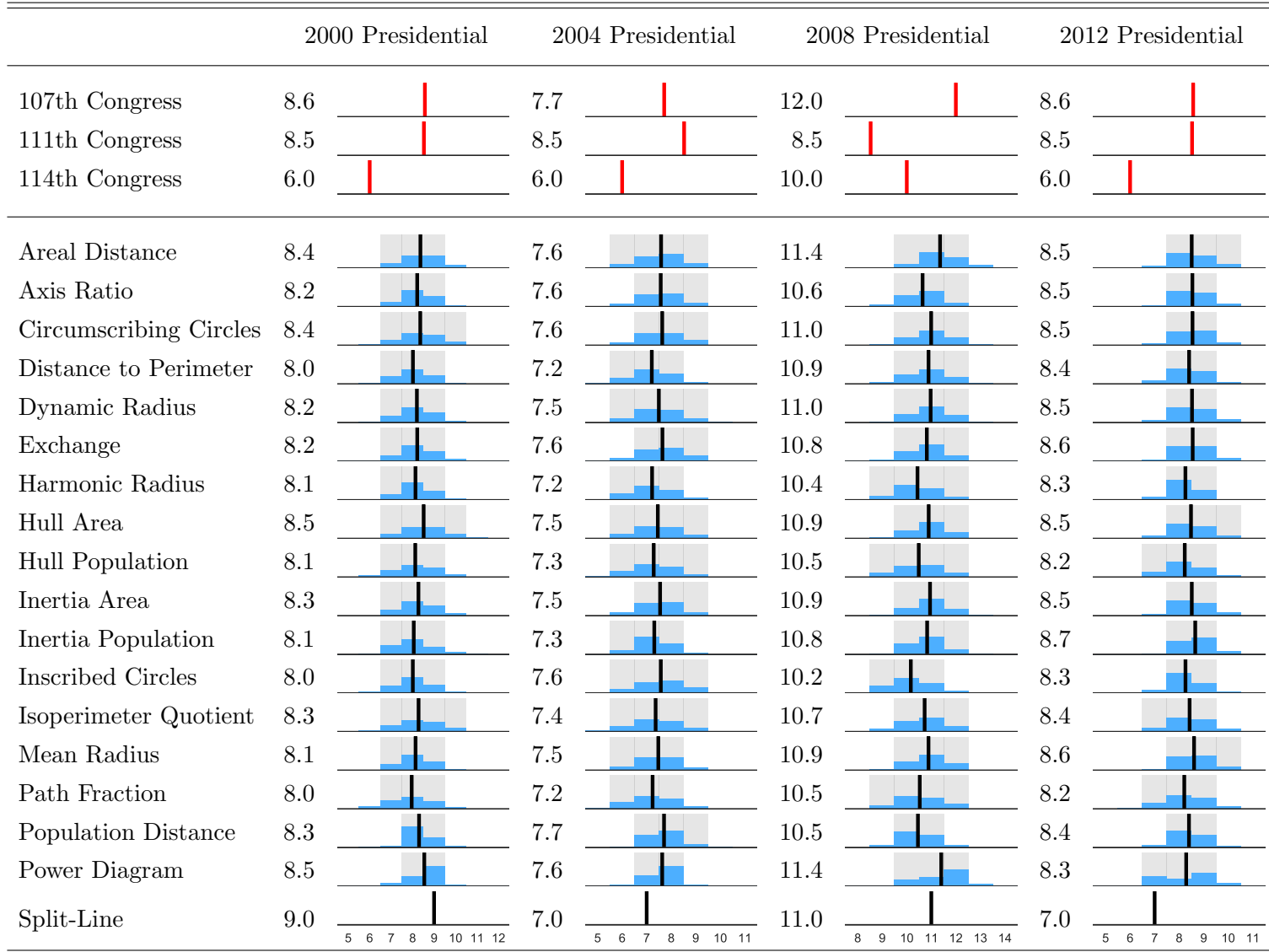


TABLE 2. Votes from presidential elections in Pennsylvania are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 18 assigned after the 2010 Census.

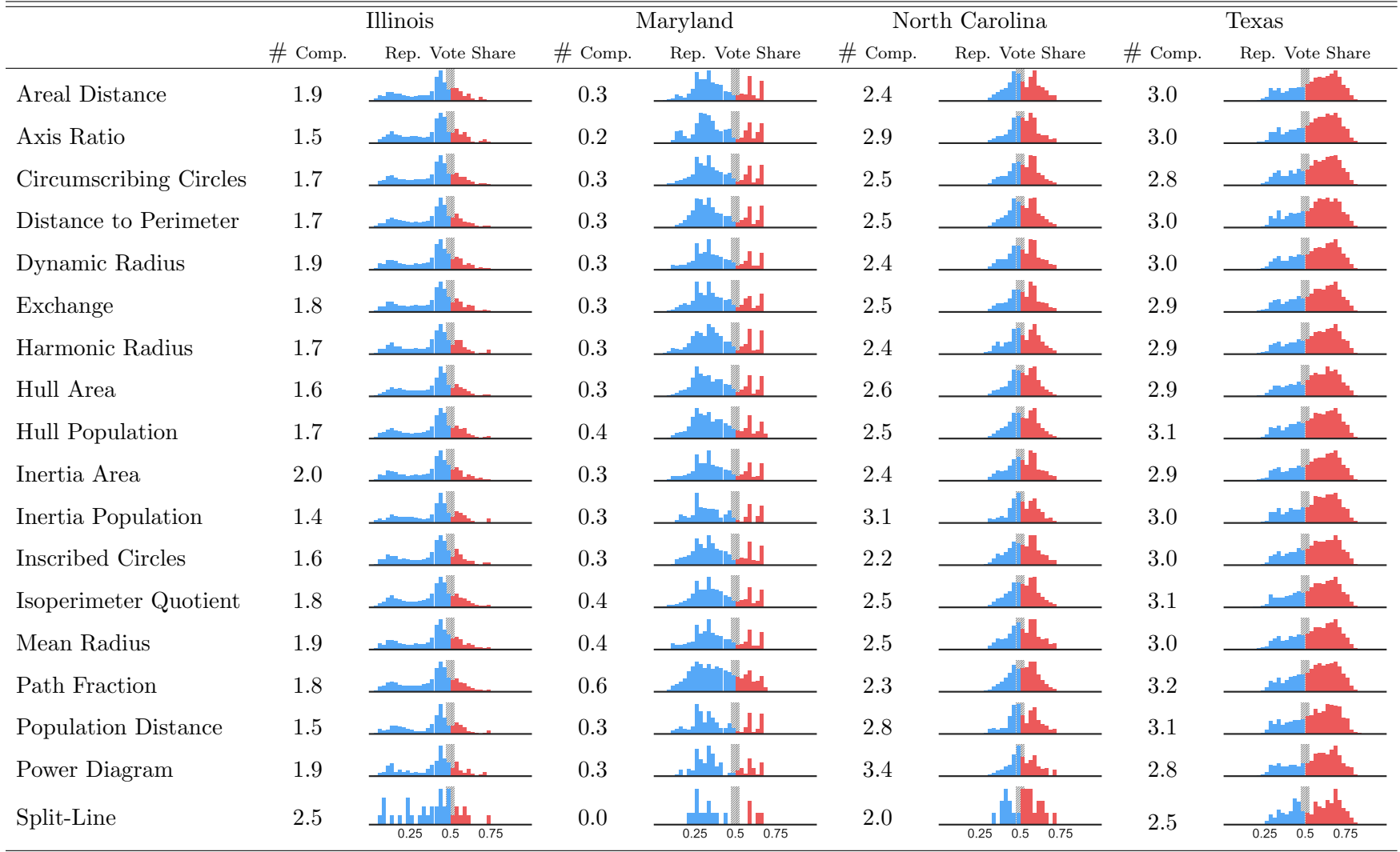


TABLE 3. The vote shares accruing to Republicans are plotted for all districts of each map, and for all available elections, leading to one distribution for each state and method. The consistency in the shapes of the distributions across methods suggests that the many methods do not differ in their treatment of the two parties. The different shapes for the four states shows the impact of political geography on partisan representation. Republican vote shares in excess of 0.5 correspond to Republican wins; the integral up to 0.5 corresponds to the Democratic seat share, as shown for Pennsylvania in Figure 2. The part of the distribution close to 0.5 are competitive races. To the left of each distribution, I tabulate the number of competitive races, calculated as the integral of the vote share distribution between 0.475 and 0.525. As for seat shares, the level of competitiveness is quite consistent across measures.

FIGURE 1. Various distinct “concepts” are associated with compactness. The circle is typically agreed to be the most compact shape, to which may be contrasted disperse, indented or dissected ones. But one may also look “within” a shape. In the middle frame, the dots are less disperse if one shifts the cells from the thick black bounds to the dotted ones.

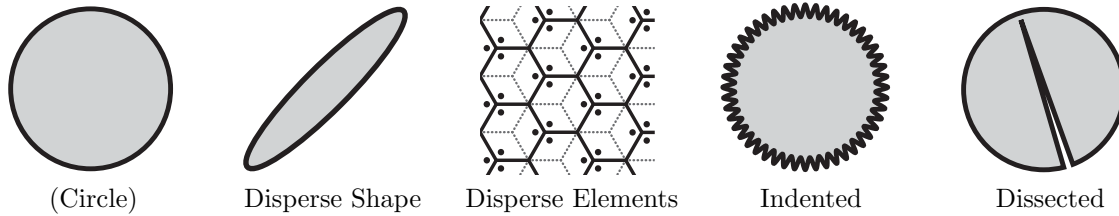


FIGURE 2. Building blocks of compactness measures, on Pennsylvania’s 7th congressional district. From these derived shapes and lengths, a great number of measures may be defined.

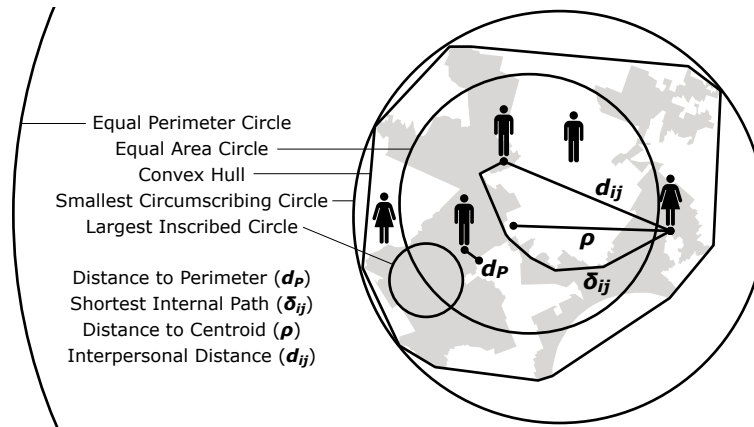


FIGURE 3. Representative districting plans of Pennsylvania, for various metrics. The treatment of Pittsburgh, halfway down in the western part of the state, evidences how optimizing according to different definitions of compactness results in different treatment of cities.

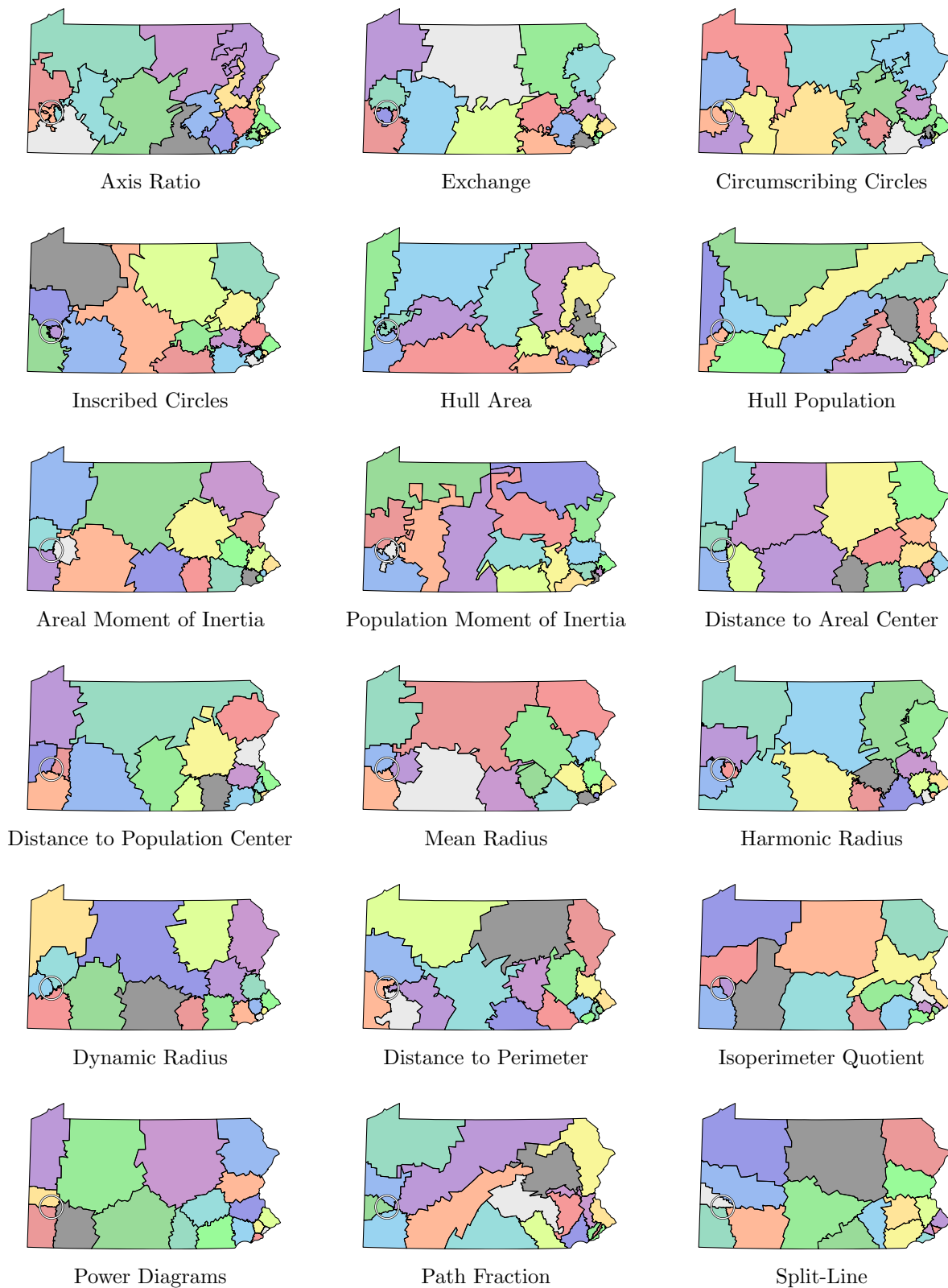
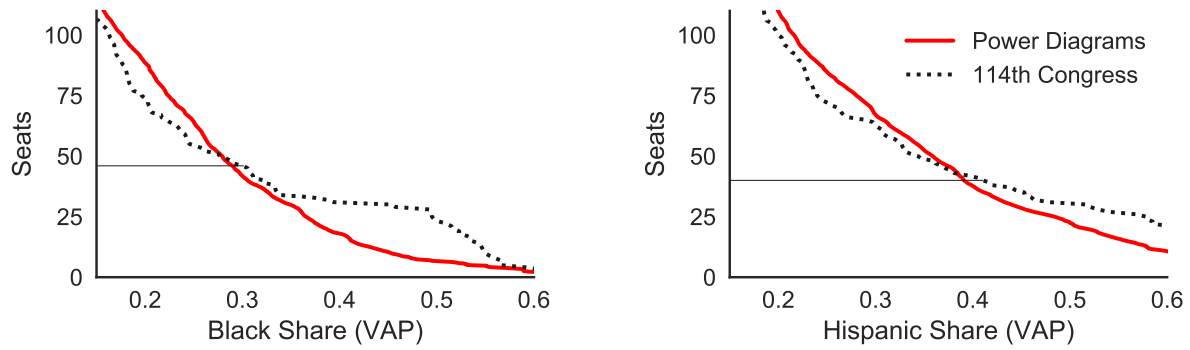


FIGURE 4. Presented are the number of districts whose population exceeds the shown thresholds of Black or Hispanic voting age population (VAP). For example, there are 74 districts whose population is at least 20% Black, and 45 districts whose population is at least 30% Black. The composition of the 115th Congress, with 46 Black and 40 Hispanic lawmakers, is represented by the thin horizontal lines.



**APPENDICES FOR:
REVIVING LEGISLATIVE AVENUES FOR GERRYMANDERING
REFORM WITH A FLEXIBLE, AUTOMATED TOOL**

JAMES SAXON

Harris School of Public Policy and Center for Spatial Data Science, University of Chicago

APPENDIX A. EXPECTED SEATS FOR ADDITIONAL STATES

This appendix presents the numbers of seats expected to accrue to each party, for several elections in nine additional states: Florida, Illinois, Louisiana, Maryland, Minnesota, North Carolina, Tennessee, Texas, and Wisconsin. These are shown in Tables [A.1-A.9](#)

E-mail address: jsaxon@uchicago.edu.

Date: August 5, 2019.

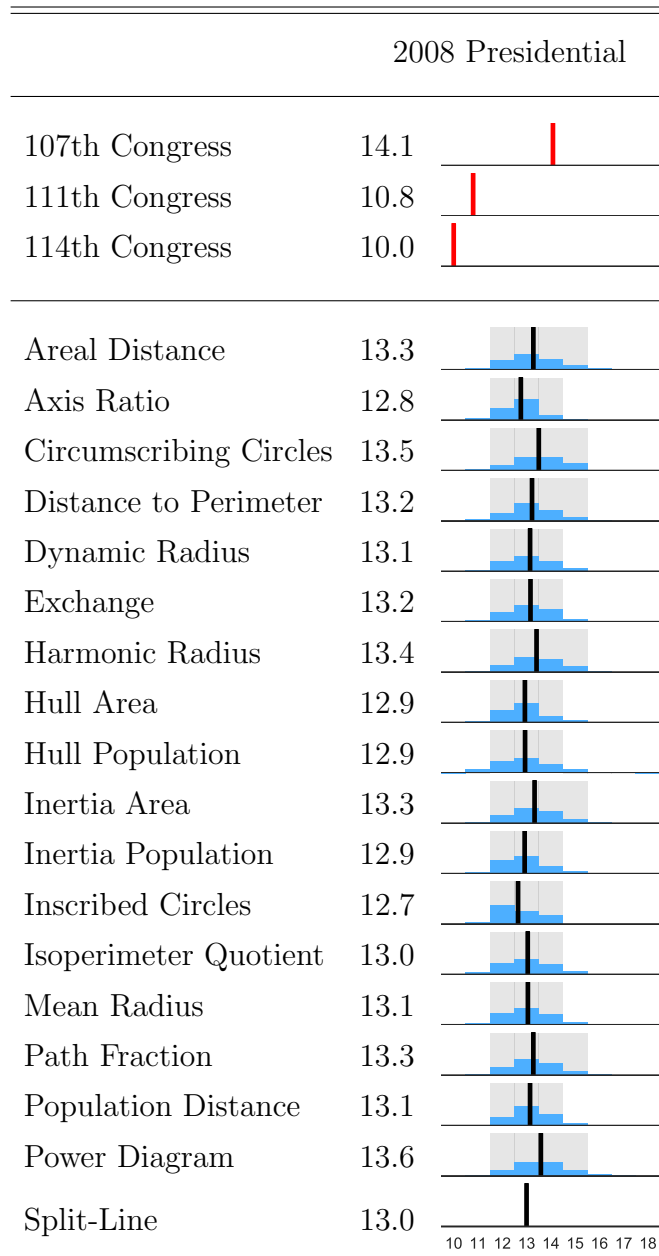


TABLE A.1. Votes from presidential elections in Florida are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 27 assigned after the 2010 Census.

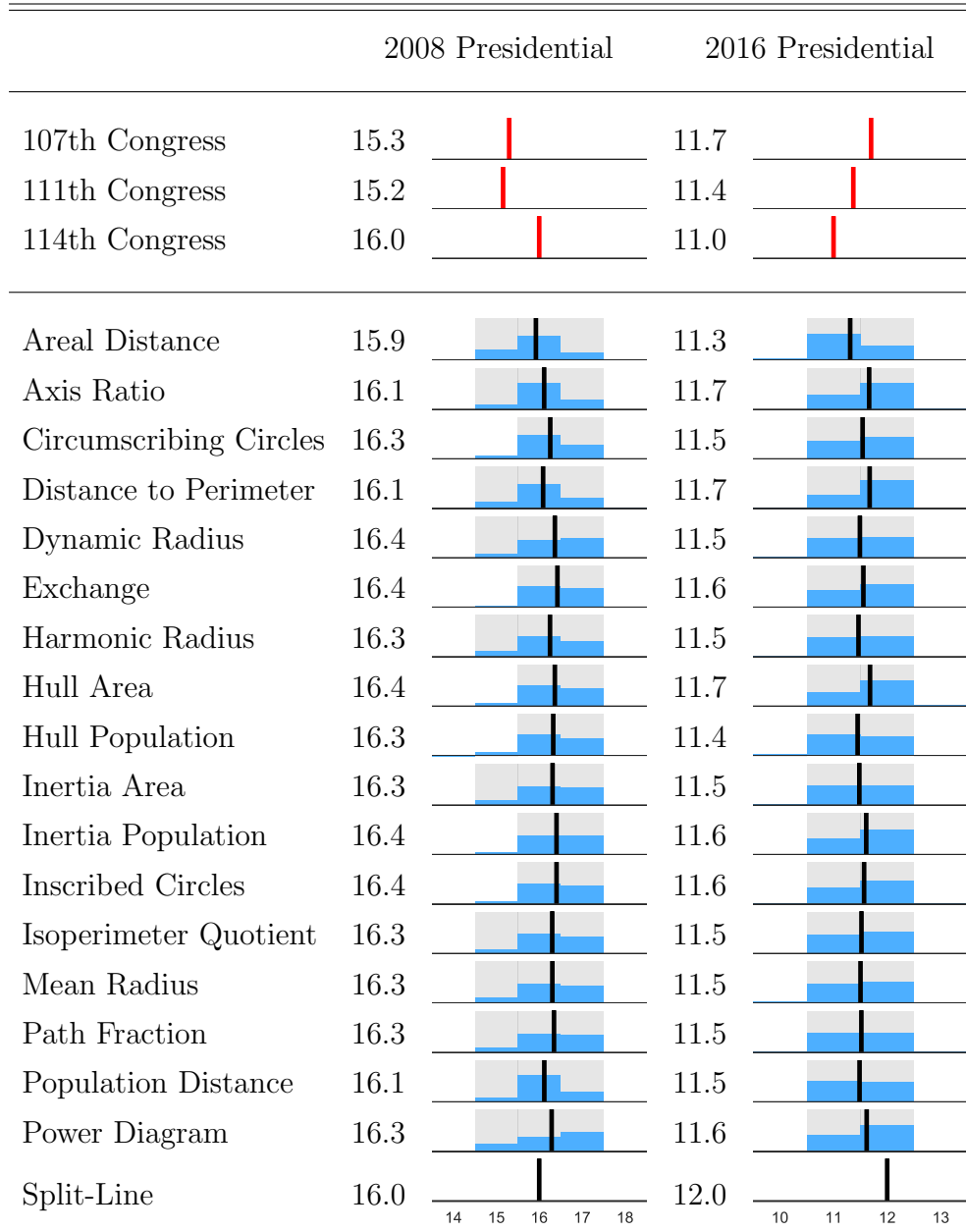


TABLE A.2. Votes from presidential elections in Illinois are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 18 assigned after the 2010 Census.

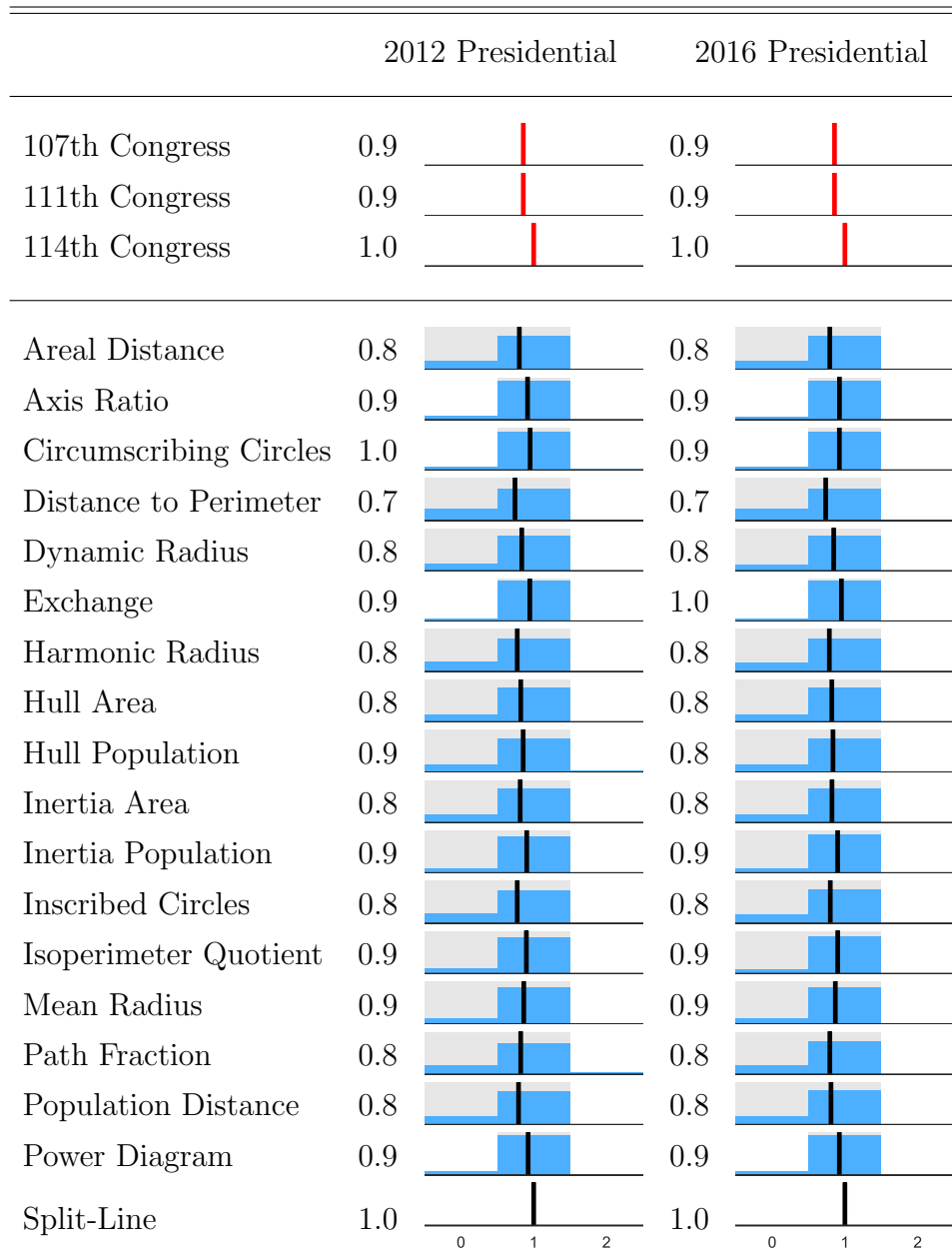


TABLE A.3. Votes from presidential elections in Louisiana are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 6 assigned after the 2010 Census.

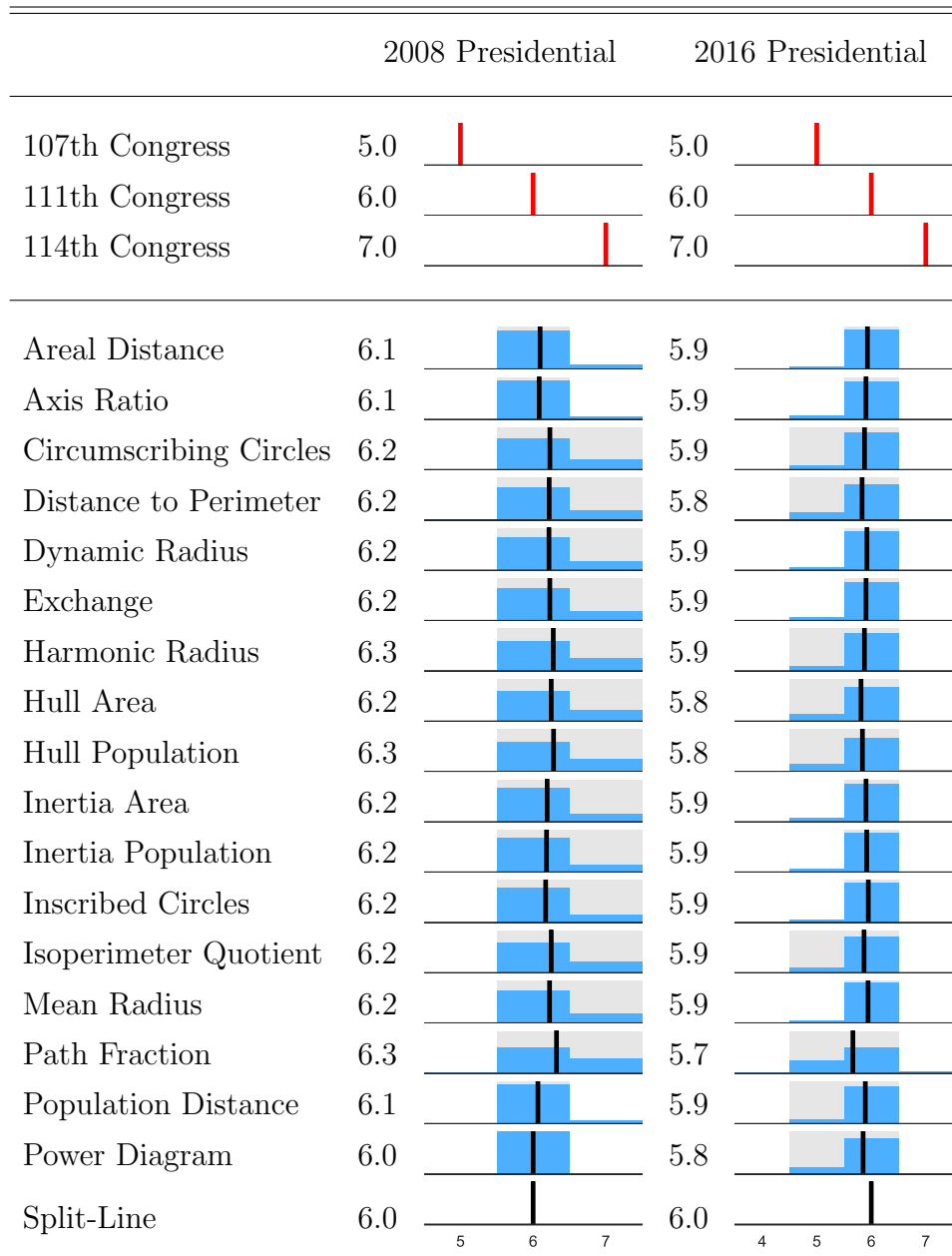


TABLE A.4. Votes from presidential elections in Maryland are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 8 assigned after the 2010 Census.

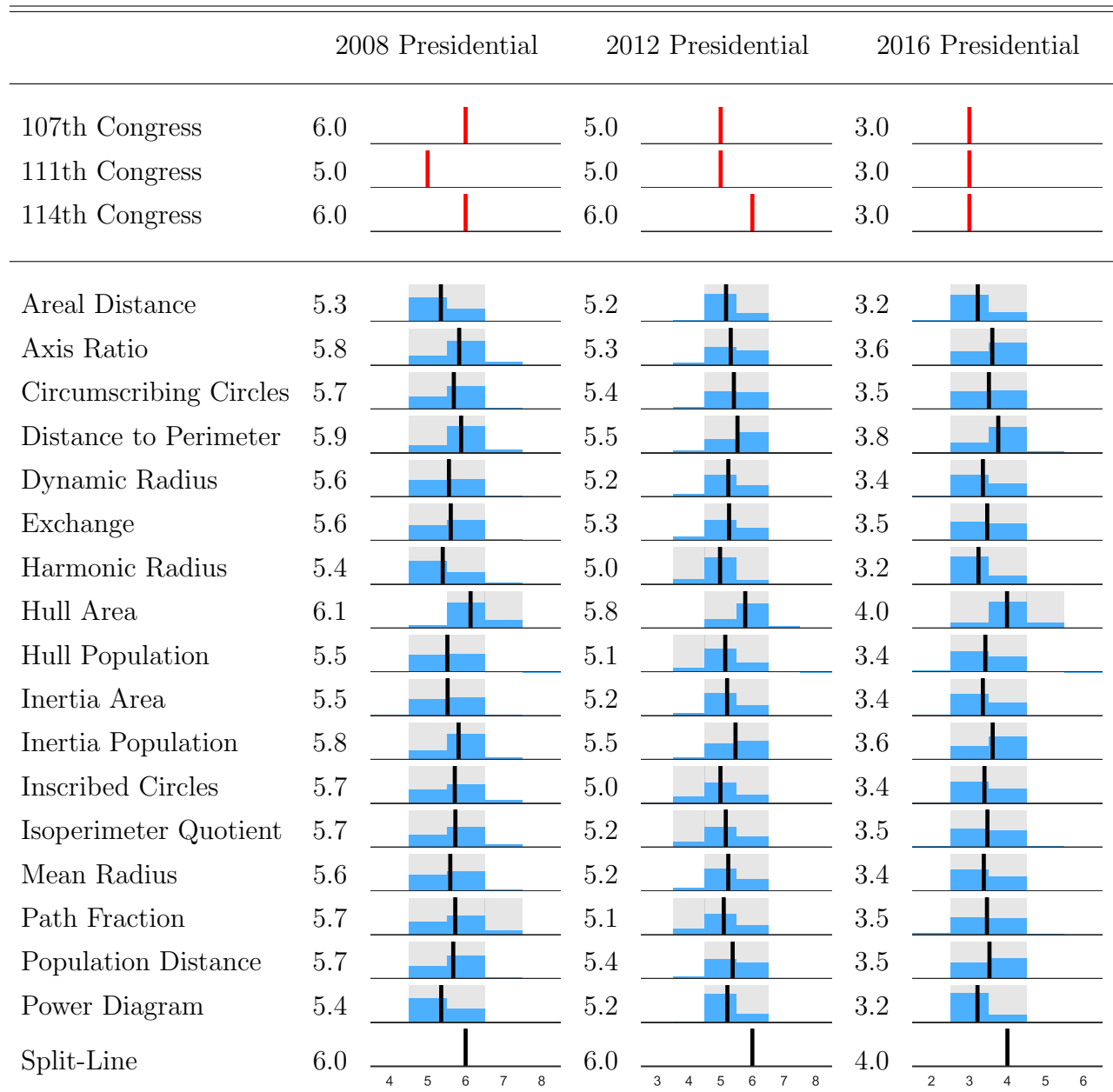


TABLE A.5. Votes from presidential elections in Minnesota are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 8 assigned after the 2010 Census.

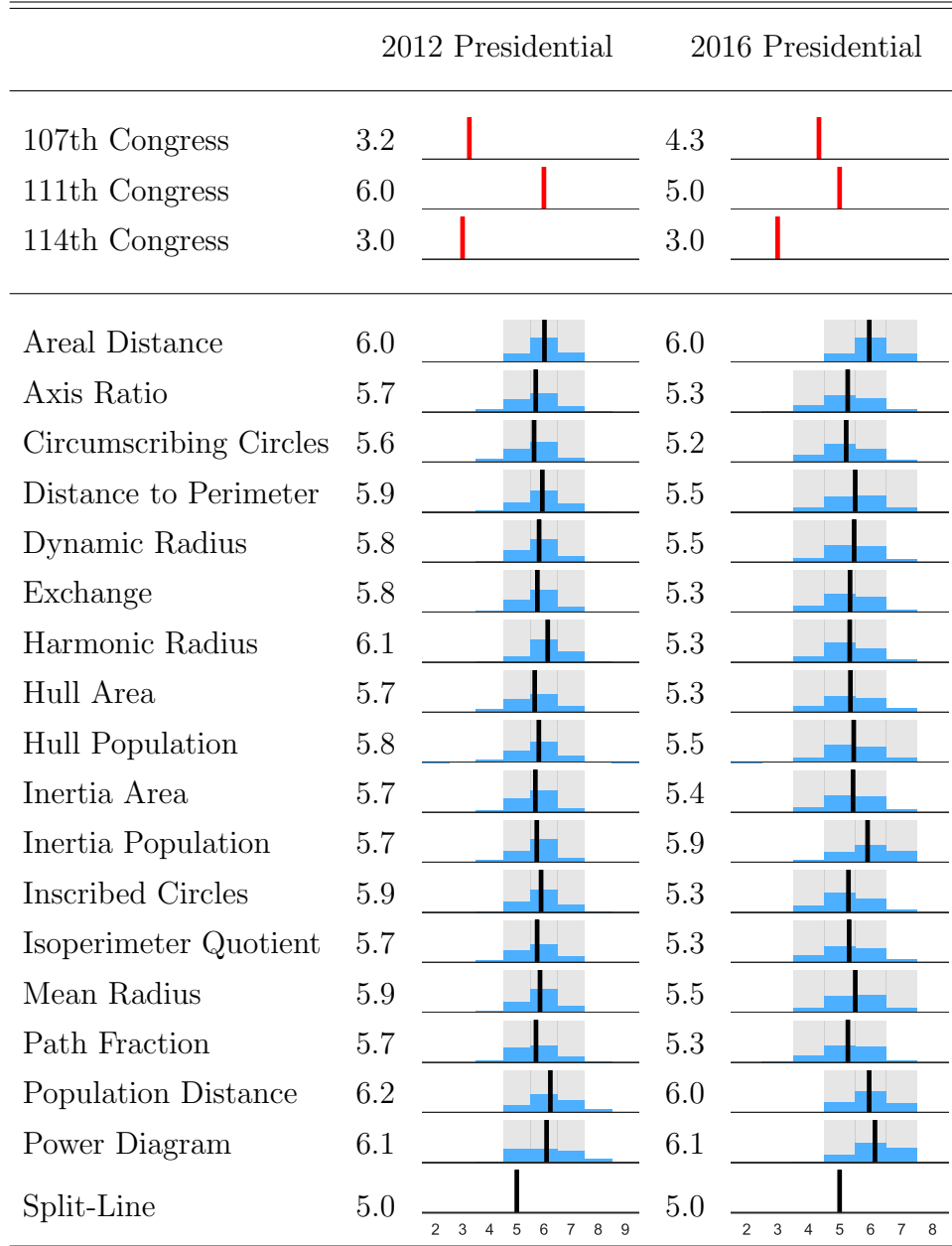


TABLE A.6. Votes from presidential elections in North Carolina are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 13 assigned after the 2010 Census.

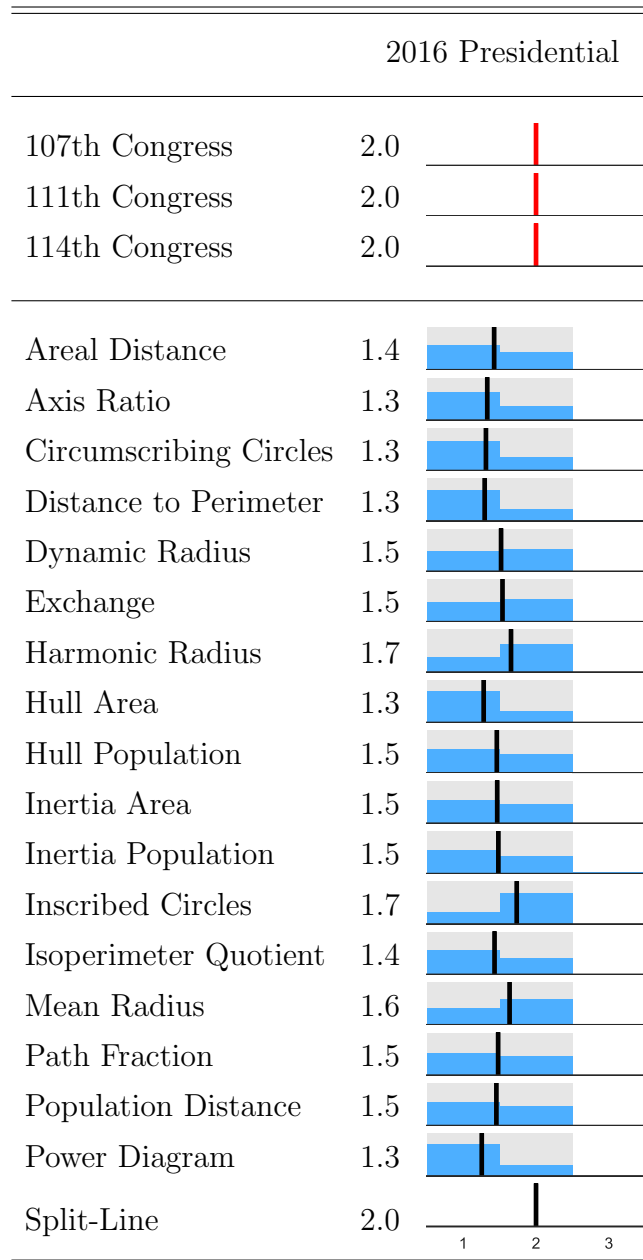


TABLE A.7. Votes from presidential elections in Tennessee are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 9 assigned after the 2010 Census.

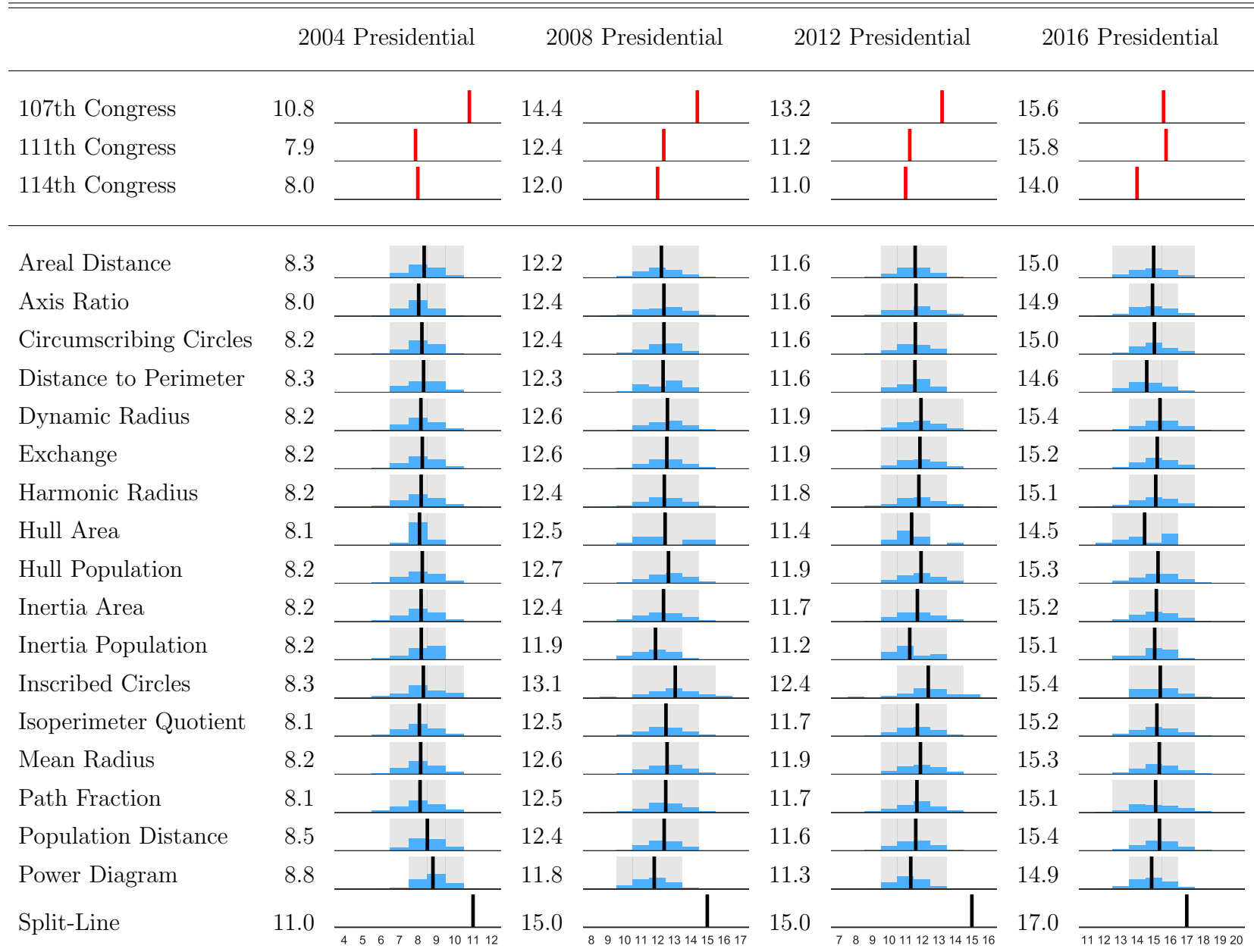


TABLE A.8. Votes from presidential elections in Texas are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 36 assigned after the 2010 Census.

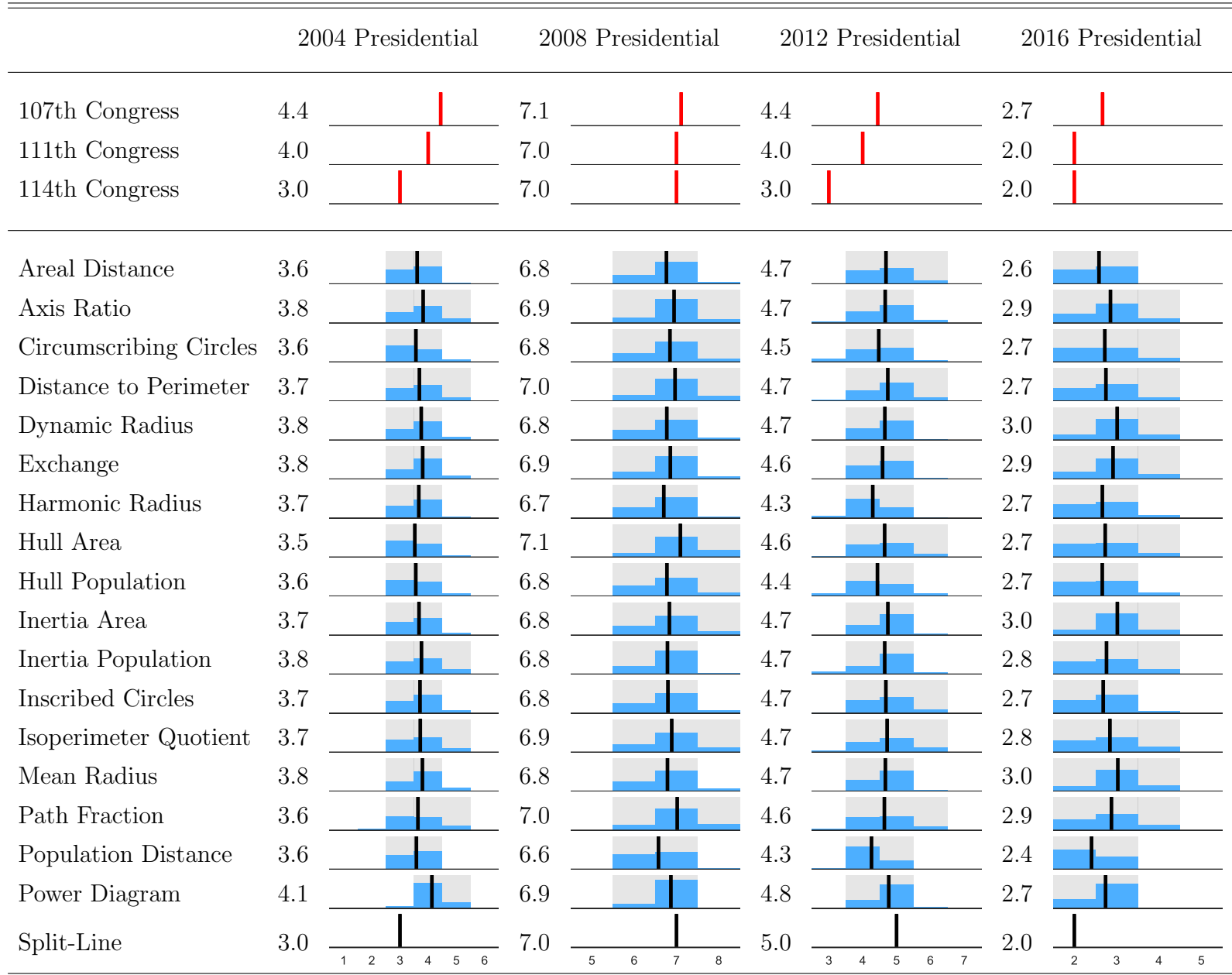


TABLE A.9. Votes from presidential elections in Wisconsin are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 8 assigned after the 2010 Census.

APPENDIX B. AUTOMATED DISTRICTING PROCEDURES

Automation has long held promise as a “solution” to gerrymandering. As early as 1961, [Vickrey](#) wrote that “elimination of gerrymandering would seem to require the establishment of an automatic and impersonal procedure for carrying out a redistricting.” By just 1963, [Weaver and Hess](#) began applying infrastructure built for the “warehouse problem” to districting in small cases (Delaware).

In the decades since, political observers presumed that with the political will, the districting problem would wilt before the burgeoning capacity of computing. In *Karcher v. Daggett* for instance, Brennan opined optimistically that “rapid advances in computer technology [...] make it relatively simple to draw contiguous districts of equal population and at the same time to further whatever secondary goals the State has.” (462 U.S. 725, 1983) This has taken longer than expected. Districting is exactly the type of (NP-hard) categorization problem that computers struggle with. So while computers quickly proved useful for tabulation and error checks, humans have until very recently performed much of the partitioning by hand. ([Altman et al. 2005](#)) Yet with appropriate metaheuristics and approximations, solutions are now in reach. In the past few years, a number of scholars have implemented serviceable automated procedures for individual metrics or algorithms. ([Chen and Rodden 2013, 2015](#); [Cho and Liu 2016](#); [Chou et al. 2012](#); [Fryer and Holden 2011](#); [Li et al. 2014](#); [Spann et al. 2007](#))

It is critical to recognize that none of the software here terminates with an exact solution – *the* optimal result – and that the software proposed in Section 3.2 does not either.¹ In other words, there is no guarantee that the partition generated corresponds to the highest possible value of the objective function. Set in the context of the partitioning of Pennsylvania discussed earlier, none of the methods here consider every one of the 10^{3800} possibilities. That is the nature of NP-hard problems. Instead, the algorithms begin are seeded with an initial configuration and iteratively move towards a “better” solutions. By initializing the algorithm many times with different starting points, it produces a variety of different “good” outputs – what I will call a “population” of maps. [Fifield et al. \(2017\)](#) criticize this approach and have sought to situate it on firmer theoretical foundations using a Markov Chain Monte Carlo algorithm. But they demonstrate full-state simulation only for New Hampshire, and generate local modifications to the districting plan in Pennsylvania. To be clear about the statistical meaning of this project, the initialization (Section C.1 is a bona fide random draw of points (census tracts) in the state; the final output of the optimization is of course *not* random. Depending on the stability of the minima, a few maps may be represented many times. Independent seeds may have lead to similar or even identical results.

[Altman and McDonald](#) published the first modern salvo at a comprehensive “solution” to automated districting in 2011, with BARD (Better Automated ReDistricting). They implemented several objective functions and metaheuristic search procedures but did not demonstrate the results from the program and claimed only to have had “moderate success with moderate size plans.” The package has not been maintained, but [Liu et al. \(2016\)](#) did attempt to use it and found it slow. The issues seem to have been the choice of language (R), inefficient objective functions, and the use of a contiguity score instead of a constraint.

The same year that Altman and McDonald produced BARD, [Fryer and Holden \(2011\)](#) proposed to evaluate non-compactness via a “relative proximity index” (RPI). They defined the RPI as the average distance between people in a state’s districts, divided by the average

¹Excepted is the split-line algorithm, which generates a unique solution, but does not make any claims about “optimality” by any measure.

distance for an optimal plan. Assuming that an “optimal plan” would (i) respect anonymity, (ii) minimize distances between voters and (iii) be scale invariant, they prove that it is a power diagram and implement an algorithm to generate solutions (see Section 2.2.1).

In the last several years, Chen and Rodden (2013; 2015) and Cho and Liu (2016) have proposed to measure gerrymandering by comparing a population of “potential plans” to enacted ones. These too have required functional algorithms. Chen and Rodden implemented a fairly straightforward algorithm, beginning from random seeds, merging cells to form an (agglomerative) initial solution, and then trading neighbors based on proximity until the population equality is satisfied. Their initial analysis focussed on Florida, but they did apply the algorithm to twenty states, and Chen and Cottrell subsequently applied it to the full country. (2016)

Cho and Liu implemented a genetic algorithm, but allow that their crossover mechanism is “sufficiently disruptive” that it is used infrequently, and may not be much better than re-initialization. It is thus effectively a GRASP algorithm: greedy with random mutations. What distinguishes their work is the care given to the preservation of contiguity and the diversity of mutations that they allow. In short, contiguity is built-in – it cannot be violated. Given a set of cells to be removed from a region, their algorithm checks that the set’s external neighbors in the source region are themselves connected. This purely-local check only works however, because of their choices to preclude holes in their plans and use queen instead of rook weights for contiguity.² Though they discuss the extensibility of their framework, they implement only the IPQ measure, which is computationally extremely simple. They use the National Center for Supercomputing Applications to generate millions of solutions, which is impressive but statistically unnecessary and hampers reproducibility. They only demonstrate plans for Maryland and do not give a good sense of the overall quality of their plans.

Kimbrough et al. (2011) and Chou et al. (2012, 2014) also implement a genetic algorithm with no crossover, to optimize an extremely simple measure of compactness: the maximum intradistrict distance. The crux of their series of papers is a lovely suggestion to use Interactive Evolutionary Computation (IEC) with what they call solution pluralism. In other words: they ask actual people to compare plans, and use these to validate the performance of the IPQ, moment of inertia, and perimeter measures. They do this with subjects both in-person and through Amazon Mechanical Turk. This work would then ideally lead to a collection of solutions for debate.

Of course, the regionalization literature extends far beyond districting. Duque et al. (2011) provides an outstanding review of regionalization methods, with a useful classification of heuristics. Though this project focusses on trading optimizations, the graph theoretic and hierarchical/agglomerative strategies he reviews could usefully extend this work. From a practical perspective, Li et al. (2013, 2014) propose a generic tool for compact regionalization with goals similar to the present work. Their tool optimizes a normalized moment of inertia, and they compare the performance of greedy, GRASP, and tabu searches. They obtain their best results from the randomization in GRASP.

Finally, a number of regionalization algorithms are available in clusterPy (Duque et al. 2011). Unfortunately, the feature set is not well-suited to the districting problem. First, the objective function is fixed (maximum in-region homogeneity on intensive variables). Further, though many of the included algorithms allow the user to specify the number of regions, they

²With “queen” contiguity, elements touching at a single point are considered contiguous, whereas “rook” quantity requires the contiguous elements to share non-zero perimeter. Cho et al state that their choice of Queen weights and no holes is legal rather than computational, but do not provide sources for this.

do not provide the possibility of enforcing a minimum population deviation among the regions. The notable exception is the max- p algorithm (Duque et al. 2012), which is also available in PySAL. (Rey and Anselin 2010) Though max- p generates the number of regions endogenously, one can easily obtain the desired number by requiring each district’s population to match the target. The algorithm does not intrinsically privilege geographical compactness, but this can be achieved by providing latitude and longitude as clustering variables.

APPENDIX C. THE $C4$ SOFTWARE

$C4$ is a multi-objective application for contiguity-constrained clustering. It is implemented in c++, with bindings to python with cython. This allows it to draw on the strengths of the two languages: c++ for speed and python for easy configurability and post-processing (mapping, analysis, etc.). I use PostGIS/Postgresql geographic database software to prepare the inputs in two ways: I simplify the boundaries between census tracts, and I extract the contiguity matrix among the cells in a state. This pre-processing stage can be cached, so there is no dependence on a private database. The software is open-sourced and freely available on [GitHub](#).

The core code is implemented as three classes: (1) a universe which owns a collection of (2) regions and (3) cells. In the present application, regions are congressional districts and cells are census tracts. Speed requires explicit memory management: the cells are instantiated once by the universe and never copied. Each region maintains a vectors of pointers to the cells currently assigned to it, and additional vectors for the cells along its internal and external borders. The cells hold their properties (location, area, perimeter, population, etc.) as well as pointers to their neighbors.

To start, the user loads the cells and adjacencies, and starts an initialization round that creates a contiguous solution from which to iterate. The iterative search then begins, trading cells between regions to equalize populations and improve one of the compactness metrics described above. Upon convergence, the program returns a list of assignments of cells (census tracts) to regions (congressional districts).

C.1. Initialization strategies. A number of initialization strategies are implemented. The split-line and power diagram algorithms discussed earlier are stand-alone, and can be used to initialize a statewide plan. A simple k -means clustering approach is also provided, as well as with random contiguous growth. An existing plan can also be loaded.

However, for the studies presented in this paper, initialization consists simply of a random draw of a cell (census tract) for each region. This ensures that the single strategy under study is completely responsible for the outcomes presented.

C.2. Contiguity. Implementing contiguity as a constraint instead of an objective significantly improves performance. To do this, the contiguity graph of the state is extracted from its topology in Postgres. In this graph, each cell (census tract) is a node, and edges are drawn between adjacent cells (i.e., neighbors). To preserve contiguity in the search, the graph must be modified in two ways:

(1) The statewide graph must be connected in the first place. $C4$ provides an automated procedure that connects islands to their closest neighbors until the graph is connected, but it also allows the user to specify explicit connections. In practice, I connect islands and disconnected subgraphs to land based on bridges, ferry routes, and shared jurisdictions. For example, the Eastern Shore of Virginia is connected to Virginia Beach by the Bay Bridge, and

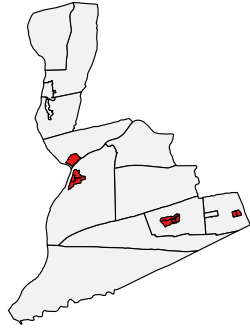


FIGURE C. Census tracts of Northumberland County, Pennsylvania are shown with enclaves highlighted. For a trading algorithm to work consistently with a contiguity requirement, enclaves and islands must be assigned to their external neighbor or nearest land neighbor.

San Nicolas Island, California is connected to Naval Base Ventura County, of which it is a detachment.

(2) “Enclave” census tracts must be assigned to their external neighbor. An example of this is shown in Figure C. Without merging the highlighted cells, it would be impossible to move their outer neighbors without violating contiguity. This argument also applies to coastal tracts that are not enclaves of another tract, but whose entire land border is shared with a single tract. The topology is different, but the graph structure is the same: they are across a “cut vertex” from the main body of the district. It is worth noting that this procedure modifies slightly the definitions described above: the compactness scores and optimizations are evaluated on this merged (or “pruned”) topology.

After these modifications, any move can be required to preserve the source region’s contiguity check: a cell’s neighbors in the source region must be members of a connected subgraph of the source region. This check resembles [Cho and Liu’s](#) local check, but it differs in that the neighbors do not have to be directly connected. If they are not, the search extends through the neighbors’ neighbors until it is shown that the removal of the cell would result in disconnected subgraphs in the source region. The result of this distinction is that both rook and queen weights are allowed, as are holes. In practice, I have used rook contiguity.

C.3. Hill Climbing. Each iteration of C_4 consists of a loop over regions. For each region, a loop over the neighboring (external) cells identifies those cells whose removal from their current region would not break that region’s contiguity. For each such cell, the change in the combined spatial and population objective function from reassignment is evaluated, as described in Section 3.1. If, among all possible moves for a region, the best one is an improvement, that cell is removed from the source and added to the destination.

C.4. Metaheuristics. To escape local minima, I have implemented several of the touchstone metaheuristic strategies – greedy, GRASP ([Feo and Resende 1989](#)), and tabu lists ([Glover 1989](#)) – but have found in practice that naïve greedy works. However, I have also implemented a ‘de-stranding’ procedure and a high-level ‘restart’ procedure, that do prove important.

C.4.1. De-stranding. “Strands” or “tentacles” occur somewhat frequently with the inscribed and circumscribing circle methods, where the objective function does not prioritize closer over further cells unless they affect the reference circle. Other algorithms occasionally also fall into a Catch-22 where the end of a long strand is not an “optimal candidate” for a trade, but no cell but the last one can be moved without violating contiguity. Since the last cell is never removed, the strand cannot be removed. I target this behavior directly, by including a configurable “de-strand-ing” procedure that trims away subgraphs connected to the main region at a single node (cell). This method is admittedly ad hoc, but as long strands and

singly-connected subgraphs are not consistent with any of the methods near global optima, I believe its sparse use is justifiable.

C.4.2. *Split-restart.* When an optimization does not improve for a configurable number of cycles, it terminates. If activated, the split-restart procedure then (a) splits the district with the worst score on the current metric, and then (b) merges a (random) district with one of its neighbors. This usually results in one district with twice the target population and two with half the target. The interest of this is to give the system a very large shock that nevertheless leaves it in a fairly good starting solution to generate a new partitioning. The shocks are large enough to yield distinct maps in each cycle, but leave the better (more-compact) districts intact. The starting solutions thus typically improve with each cycle. The method is philosophically appealing since it is entirely based on the reference measure; unlike “de-stranding,” it does not import outside prejudices about “good” behavior.

C.5. *Approximations.* Despite significant care to implement the objective functions efficiently, a number of approximations are required to achieve adequate performance. Foremost among these is the use of cell centroids in calculating the convex hull and circumscribing circles, and the various distances (to the perimeter or center). For the circumscribing circles, an approximation of the perimeter is made from the topology, tracing the trijunctions of census tracts along the border. This method was first suggested by [Schwartzberg \(1966\)](#) for a variant of the IPQ measure. For the fraction of shortest internal paths that are themselves in the district, I constrain the path to run between centroids of adjacent cells, in order to simplify the network problem.³

APPENDIX D. RESULTS WITH STATE LEVEL RACES

This paper presents seat share results for multiple (presidential) elections per state, when data permit. This shows that with geographically realistic vote distributions, the fundamental result on the consistency of seat shares between methods is robust to dramatic changes in statewide vote shares. In Illinois, Minnesota, and Wisconsin for instance, Barack Obama garnered far more votes in 2008 than Hillary Clinton did in 2016, but the seat shares within years remain consistent across districting methods (Tables [A.2](#), [A.5](#), and [A.9](#)). Using national races ensures that all races are contested by competitive candidates, but it is an approximation because local candidates might be better able to respond to local preferences.

Table [D](#) presents aggregated seat shares in Texas, replacing presidential contests (Appendix [A](#), Table [A.8](#)) with elections to the US Senate. Texas is presented simply because many years of data are readily available. As in Illinois between 2008 and 2016, the vote shares vacillate wildly in the four elections shown. John Cornyn garnered 64% of the two-party share in 2014 while Ted Cruz won just 51% in 2018. Despite this swing, the table shows that the empirical consistency in seat shares among methods is preserved when shifting to state level races.

³The exact solution, of crossing a visibility graph is not feasible since the graph must be reconstructed for every proposed move, and the shortest paths reevaluated.

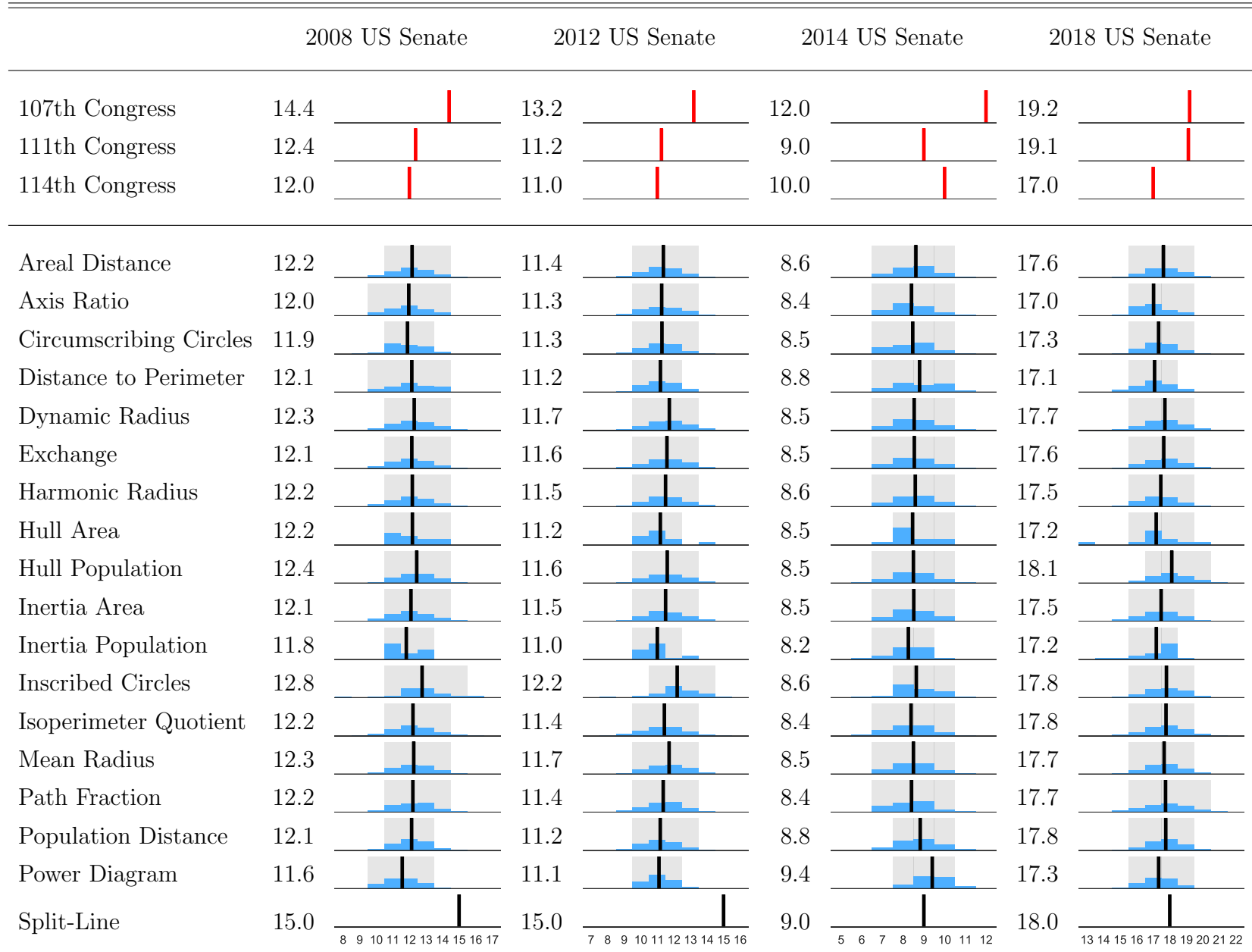


TABLE D. Votes from presidential elections in Texas are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 36 assigned after the 2010 Census.

APPENDIX E. PRESERVING COMMUNITIES: OBJECTIVES AND EVALUATIONS

This paper focusses narrowly on the spatial compactness of Congressional districts, since it was the last objective enacted by Congress. It is not the only districting criterion, however. This Appendix illustrates how alternative objectives may be included in the framework developed in this paper. The Supreme Court has itemized traditional districting principles regularly and consistently for over half a century, as “including but not limited to compactness, contiguity, respect for political subdivisions or communities defined by actual shared interests.” (*Miller v. Johnson* 515 U.S. 900, 1995, 916)⁴ This respect for existing political subdivisions stands foremost among the other objectives.

It can be incorporated in the framework of this paper in a number of ways: as an optimization objective, as a criterion for evaluation and comparison, or as a hard requirement on permissible maps. This Appendix develops the first two strategies. (The hard binary requirements are used in Appendix F in the context of minority representation.) I define first an objective for community preservation (E.1), and then a measure of split communities (E.2), before presenting and discussing results (E.3). Note that my definitions are intended only to illustrate how such conditions might be written mathematically and algorithmically; they are by no means generally accepted or legal doctrine.

E.1. Modified objective functions. There are a number of ways of implementing a “respect for existing political subdivisions” as an objective function. The first is to add an explicit and separate measure of community preservation in the objective function for each district. This component would be weighed against compactness and equipopulation. The second strategy is to modify the compactness-based functions to account for existing boundaries. I take the second approach.

The Isoperimeter Quotient (IPQ, or Polsby-Popper) method considers the ratio of a district’s area to the square of its perimeter (see subsection 2.1.1). To account for existing political subdivisions, perimeters can be downweighted – made less costly – when they fall along the extant boundaries between subdivisions. In the context of district optimization, this will privilege district borders that coincide with existing lines. For this Appendix, I define a county-weighted IPQ by weighting perimeters along county lines by a factor of 2/3. This reweighting could be extended with overlapping communities of varying importance.

Many of the other compactness measures can be modified along similar lines. For example, for methods derived from distances among residents (power diagrams) or to the district center (moments of inertia), components of a community can be “shrunk” towards its center. For county preservation, this simply means moving the census tracts in each county towards the center of the county.

E.2. Evaluating the performance of existing maps. A common way of evaluating how well a districting plan respects existing communities is to count the number of times those communities are split between districts. This method is insensitive to variation in how “grievously” the community is divided. It may be worse to split a county in two than to shave a bit from its edge. Naïve counts also ignore the fact that some communities exceed the scale of a legislative district and *must* be divided.

⁴Consistent language can be found in *Reynolds v. Sims* (377 U.S. 533, 1964, 580-581), *Davis v. Bandemer* (478 U.S. 109, 1986, 116-117, 138-39, 162, 173), *Shaw v. Reno* (509 U.S. 630, 1993, 16, 47), *Karcher v. Daggett* (462 U.S. 725, 1983, 755), *Bush v. Vera* (517 U.S. 952, 1996, 977), or *Vieth v. Jubelirer* (541 U.S. 267, 2004, 273, 322), among many others.

I define a measure of community-splits as the likelihood of two residents of a community having the same legislator. Formally, I define this likelihood by dividing the population of shared (intersecting) county residence and Congressional representation by its maximum value. The denominator is then the lesser of the community (county) population and that of the legislative district. Denote the legislative district of individual i by $LD(i)$, his or her community by $comm(i)$, the intersection of the two by $(LD(i) \cap comm(i))$, and their populations by $p(\cdot)$. The “shared legislator” value for constituent i can then be written

$$s_i = p(LD(i) \cap comm(i)) / \min(p(LD(i)), p(comm(i))).$$

As defined, this measure lies in the range from 0 to 1. Higher values indicate better respect for existing communities. This is one definition among many that could be defined, and over which reasoned debate might prove useful.

E.3. Results for community-based objectives and criteria. Table ?? shows the likelihood of sharing a legislator with county co-residents, in three states whose 2012 Congressional districts have been extensively litigated: Maryland, North Carolina, and Pennsylvania. The first three rows of that Table show enacted plans. The division of counties has gotten worse over time in all three states. This is true even though Pennsylvania lost seats in Congress in both 2000 and 2010, and therefore has mechanically fewer divisions to make. North Carolina, on the other hand, gained one seat in 2000. On the whole, citizens share legislators with about 70% of the residents of their county, out of the maximum value already discussed.

Plans enacted for the 1992 redistricting generally outperform simulated maps in the states shown, but this has ceased to be true as the enacted plans have gotten worse. The simulated maps outperform the plans used for the 114th Congress in Maryland and North Carolina, and in Pennsylvania they are comparable. The table also includes results for maps derived with the “county-weighted IPQ” as the objective function. With respect to the nominal IPQ, this simple and conservative modification increases likelihood of co-residents sharing a legislator by a few percent, for each of the three states. By contrast, the path fraction consistently underperforms its peers. It is interesting to note that power diagram districts (which are always convex), have high path fraction, but that path districts with good path fraction may have poor interpersonal distance (the measure corresponding to the path diagram algorithm – see Section 2.2.1). The space of high-scoring districts is different between the two methods, and this affects performance on this measure of divided constituencies.

There are two conclusions to draw from this work. The substantive conclusion is that automated optimization of spatial objective functions does not significantly exacerbate the division of counties. At least in some states, compactness does better. This might not be a surprise, since counties are usually quite compact. But it shows that alternative objectives cannot reliably be depended on to absolve spatial monstrosities of their partisan intent.

The methodological point is that it is straightforward to extend the methods of this paper to districting objectives beyond compactness.

APPENDIX F. POST-SELECTION OF MINORITY MAJORITY DISTRICTS

The main text of this paper affects distributions of maps in states by changing how they are generated. It is also trivial to alter these generated distributions by selecting subsamples with specific properties. For observables correlated with the selection criteria, the distributions in the subsample will differ from the full population. This Appendix illustrates this selection method, as applied to minority representation in North Carolina.

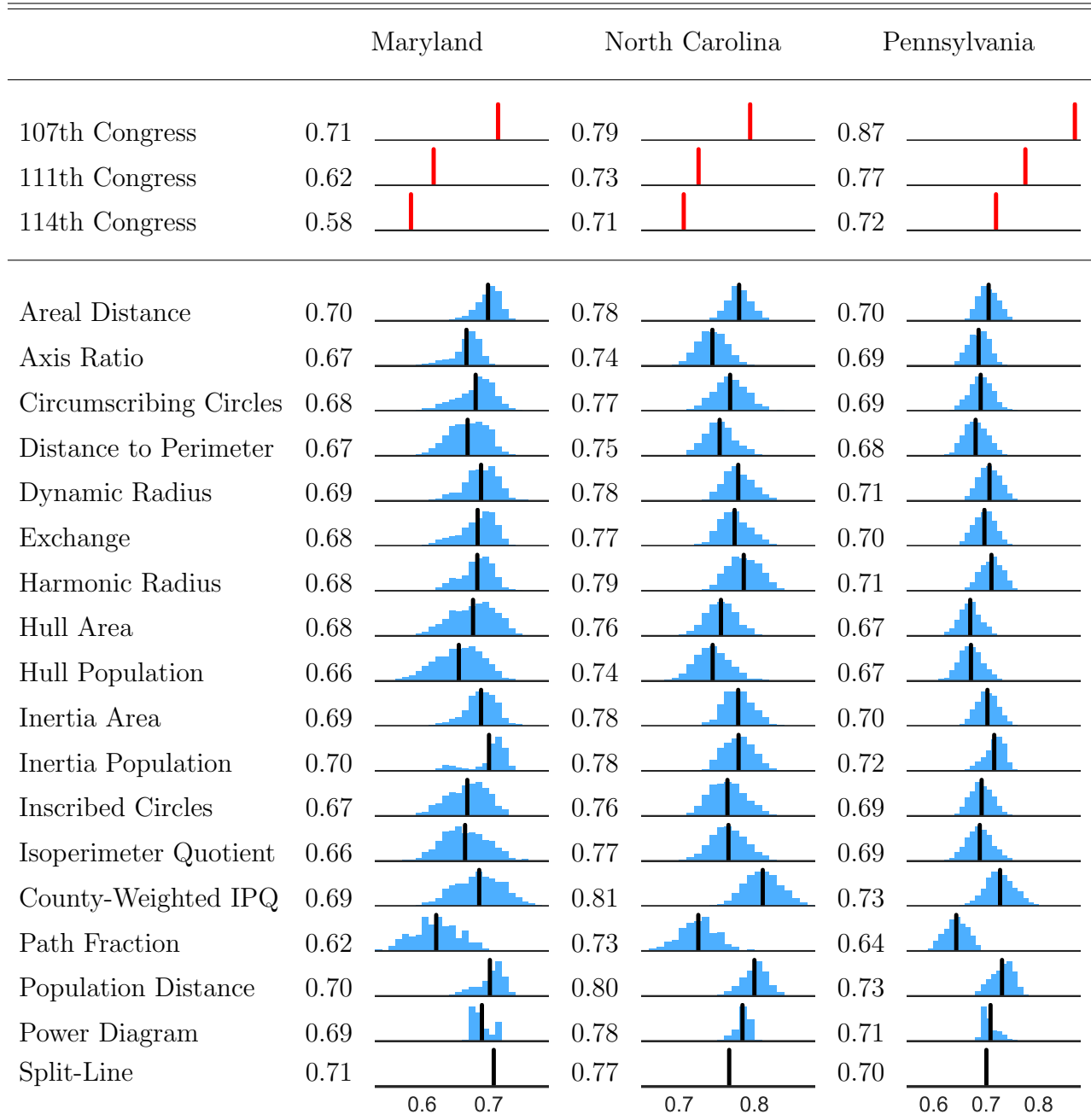


TABLE E. Probability of a citizen of a county living in the same congressional district as another randomly-selected citizen of their county, for distributions of maps derived for various compactness definitions. The “probability” is modified to account for counties larger than Congressional Districts (see text). North Carolina gained a seat after the 2000 Census, whereas Pennsylvania lost seats in both 2002 and 2012. The probabilities do not correct for this effect (first three rows). There is a progression towards more-divided counties among the enacted maps. For Maryland and North Carolina, the automated procedures outperform the enacted maps. The county-weighted isoperimeter quotient measure outperforms baseline IPQ by a few percent.

Section 5.3 showed that a compactness requirement would engender small reductions in minority seat shares, nationally. I argued that these reductions would in practice likely be more severe than indicated in the simple model. That model was based on race/ethnicity alone, and did not ensure the party composition necessary for minority candidates to be successful. How then, to ensure better outcomes for minority voters? Perhaps the easiest way is to select statewide plans that favor them. To do this, I select statewide maps from the samples generated for North Carolina, that have at least two seats where over 30% of the voting age population is Black, *and* with a Democratic two-party share over half (averaged between 2012 and 2016). This is a rough requirement, intended to match or exceed the Black seat share of the state’s current delegation. More-precise and involved selection mechanisms for minority seats could easily be concocted.

This requirement rejects a large fraction of the generated maps, including the single split-line map. One might expect that selecting on the (imperfectly) correlated observables of party and race would increase the Democratic two-party share above the 50% required to win. This might lead Democratic votes statewide to be “less efficient,” and result in their capturing fewer seats. The actual behavior depends on the spatial configurations of the maps. For example, the correlation between the Black share in the second most Black Democratic district and the number of Democratic seats is negative for power diagrams (-0.20) but positive for isoperimeter quotient (0.28). Table F shows the Democratic seat shares for this subsample. With the cuts defined, the seat distributions from the subsample are remarkably consistent with the full populations (Cf Table A.6).

For power diagrams, however, this consistency turns out to be “lucky.” The minima of the power diagram method are more stable than many of the other methods, and plans optimized with different seeds can terminate with the same geographic configuration. This means fewer substantially-different maps, and more bunching in any derived distribution. Figure F shows the share of the voting age population that is Black, in the Democratic district with the second-highest share, for maps derived using power diagrams or isoperimeter quotient. To simplify the graphic, I define party based simply on the vote shares in 2012. Requiring two Democratic districts with a voting age population more than one third Black leaves no power diagram maps at all. Lowering the threshold for the Black share to 0.32 rejects all maps with less than six Democratic seats, and raises the expected Democratic seat share. But lowering it yet again to 0.29 does the reverse: it allows the maps with four Democratic seats, and removes more of the maps that *favor* Democrats.

The other way to read this plot is that the full population of maps shows a high degree of consistency in the values simulated for a rather abstruse observable. For the IPQ and power diagram methods, the range in the Black share of the second-most Black district is largely contained between 0.25 and 0.35. The ranges are even tighter, within methods. To alter the distributions of these observables, one could of course include them as objectives in the optimization, as per Appendix E. If two Democratic districts were desired with a voting age population greater than 40% Black, intervention would be necessary at the optimization stage.

This post-selection strategy is trivial to implement for any manner of political, demographic, or spatial conditions. Selection affects other observables, including the representation of the parties, in non-trivial ways. These impacts are not necessarily consistent across methods.

APPENDIX G. A LEGAL HISTORY OF COMPACTNESS

Much of the recent academic, legal, and public discussion of gerrymandering has focussed on remedies through the Supreme Court. I suggest that this focus is misdirected: for much of

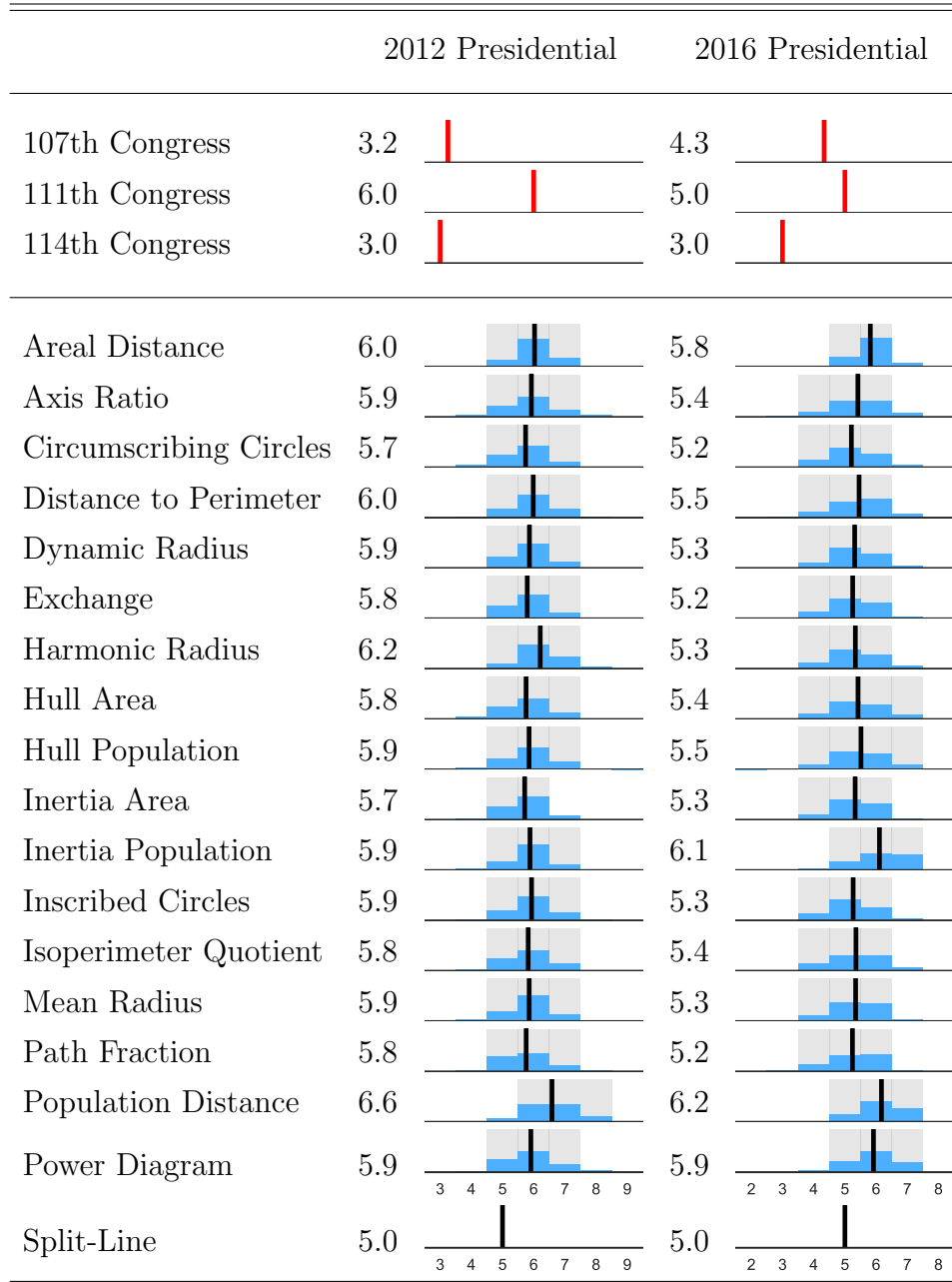
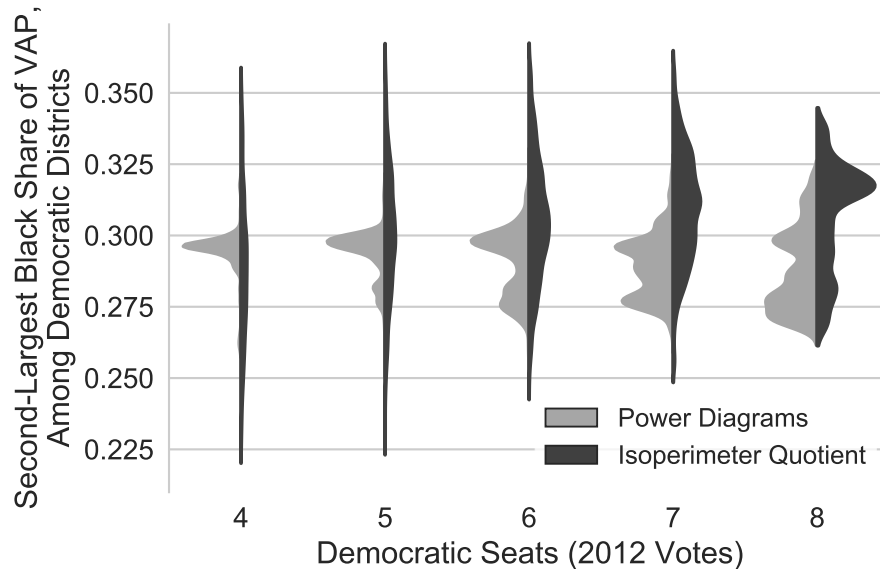


TABLE F. Votes from presidential elections in North Carolina are aggregated from precinct-level returns, into maps simulated with each algorithm or compactness metric. The seats expected to accrue to Democrats (mean across maps) are displayed numerically as well as by a solid black line. The normalized distribution of seats per metric/algorithm is shown in blue and the 10-90% range of possible seats is highlighted in gray. The same re-aggregation is performed for enacted maps used for the 107th, 111th, and 114th Congresses and shown in red. Since reapportionment shifts the number of seats per state, the entries for the 107th and 111th Congresses are the Democratic share, times the 13 assigned after the 2010 Census.

FIGURE F. The fraction of the voting age population that is black, in the second most-black Democratic district is plotted against the number of Democratic districts in the map, based on votes from 2012. Distributions are shown for maps derived with both power diagrams and the isoperimeter quotient compactness definition. Bunching in the distribution of power diagram districts makes the number of expected Democratic seats sensitive to the cut on the Black vote share. On the other hand, though the distributions differ substantially, the numerical consistency is very high between the two methods, for this (somewhat contrived) variable.



American history, debates over apportionment and districting centered instead on Congress. It is useful to recall the arc of the historical debates to situate the possible legislative and judicial uses of compactness.

G.1. Congress. The Constitution gives Congress tremendous latitude over the composition of the House and the election of its members. It stipulates that Congress reapportion the House of Representatives after each decennial census according to the relative populations of the states, but leaves the details of that procedure open. Although authority over the “Times, Places and Manner of holding Elections” is tentatively delegated to state legislatures, Congress retains supervisory power and “may at any time by Law make or alter such Regulations.” Congress is the Judge of its own elections. (U.S. Const., art. 1 §2, 4, 5)

In the state ratifying conventions, some delegates feared that Congress’s supervisory power over elections might be used to circumvent terms of office. The Federalists defended it on two main grounds. The first was practical and broadly conceded: the Federal government needed direct access to its electors, so as not to be beholden to the state legislatures. In Federalist 59, Hamilton wrote that “every government ought to contain in itself the means of its own preservation.” This was no idle worry: Rhode Island had recently boycotted the Constitutional Convention and would not ratify for another two years. The second justification reflected the same concern as the Guarantee Clause (art. 4, §4): that Congress has a responsibility to ensure just elections and equal representation to its citizens. At the Virginia convention, Madison argued that the citizens of the several states ought to be treated uniformly in the election of their representatives and remarked that “diversity [of regulations] would be obviously unjust.” Of particular interest to the present discussion, he drew for his example the

inchoate gerrymander of South Carolina, where Charleston had seized more than its fair share of members. ([Kurland, et al. 2000](#))

While the Founders clearly anticipated that the members of the House would represent geographically defined single-member districts, they did not require it in the Constitution. The ultimate scale of the House – actively defended in Federalist 9 and 10 – was similarly left to Congress in posterity. In discussing the role of the representatives in the House in Federalist 56 and 57, Madison makes it clear that each member was to represent a place-based district of “thirty thousand inhabitants” with which he would be well-acquainted. New York and Albany were each entitled to one representative, Philadelphia to two, and some of the other Pennsylvania counties to nearly one. Nevertheless, “general ticket” and multi-member districts were commonplace through the country’s history. In the first House elections, Pennsylvania, New Jersey, and Georgia elected members at large ([Cong. Quarterly 1994](#)), and the practice formally ended only in the 91st Congress. ([US Code, Title 2, §2c Amended 1967](#))

Congress began to flex its powers of self-definition with the Apportionment Act of 1842. Previous Acts had evaluated the proper scale of the House, but the 1842 debate raised the question of the form of its constituencies. It was broadly agreed that single-member districts were preferred in principle, but a number of smaller states had adopted (statewide) “general ticket” elections as a means of ensuring a “bloc vote” that would enhance their power. ([Zagarri 1987](#)) So along with the regular debate over the scale of the House, congressmen sparred over the stipulation that representatives be elected from single-member districts of contiguous territory. Critics of the proposal argued that Congress was either entirely powerless over the shape of districts or else that the supervisory powers of art. 1, §4 gave Congress the power to district – but not to direct the state legislatures *how* to district. They forecast doom should Congress overstep this bound: it would unleash war between the Federal and state governments. ([Cong. Globe 1842](#)) It is true that after the Act passed with the requirement intact, several multi-member states sent members from at-large districts, who were seated. ([Cong. Quarterly 1994](#)) But the Union held, and the states bowed to the new norm within a few years. Though the measure was not reenacted after the 1850 Census, it elicited a tame debate when it was revived and reenacted as a separate (non-apportionment) act in 1862. ([Cong. Globe 1862](#))

After subsequent censuses, Congress continued to elaborate its conception of fair representation. In 1872 it added the requirement that districts be equipopulous, which persisted in 1882. ([17 Stat. 28 1872](#); [22 Stat. 5 1882](#)) In 1901, an amendment was introduced with the stated goal of avoiding “shoestring” districts, and requiring yet further that the districts be “compact.” ([Cong. Rec. 1901](#)) That amendment passed over concerns that the new standard was ill-defined and would prove difficult of application when judging the qualifications of Representatives. The 1911 act reiterated the accumulated requirements: single member districts “composed of a contiguous and compact territory, and containing as nearly as practicable an equal number of inhabitants.” ([31 Stat. 733 1901](#); [37 Stat. 13 1911](#)) But following a botched census after the First World War, no act was passed in 1922 and a frantic Congress abandoned the requirements with little debate for the Apportionment Act of 1929. ([Cong. Rec. 1929](#)) The Supreme Court ruled in [Wood v. Broom](#) (287 U.S. 1, 1932) that Congress’s refusal to reiterate these rules amounted to their annulment.

In the aftermath of the Court’s initial assumption of responsibility for apportionment (below), Congress failed to reassert itself. In 1967 the House and Senate revived the contiguity and compactness debate, and passed a bill requiring them along with equipopulation at the

10% level. The measure failed after its second conference over disagreements on the time allowed to the states for its implementation. In its place, Congress required only that members be elected from single-member districts, which is today the only federal statute on the form of districts. (Cong. Quarterly 1968; US Code, Title 2, §2c Amended 1967)

The interest of these proceedings is that Congress has tremendous powers over the scale of the House and the form of its constituencies – and that it has exercised these powers throughout history. The statutory requirements of compactness, contiguity, and equality of population have been lost, but Congress would be well within the letter and spirit of its constitutional mandate to revive them. Should it do so, the Court would have very little recourse since under art. 1, §5, “Each House shall be the Judge of the Elections, Returns and Qualifications of its own Members.”

G.2. The Courts. Congress’ retreat was the Court’s ascendancy. This proceeded in two stages: the recognition of the justiciability first of malapportionment and then of political gerrymandering.

G.2.1. The Reapportionment Revolution. Rapid urbanization of the Country through the early 1900s paired with unresponsive state legislators resulted in galling malapportionment of legislative districts by mid-century. After refusing to enter the “political thicket” of apportionment in 1946 with *Colegrove v. Green*, the Court dramatically revised the division of judicial and legislative authority with *Baker v. Carr* (369 U.S. 186, 1962). Ruling on a badly-malapportioned Tennessee statehouse, Brennan wrote for the majority that the Court had jurisdiction on equal protection grounds, and further that the question was justiciable and *not* a “political question” per *Colegrove*. He classified political questions as those that are *inter alia* constitutionally committed to another coequal branch of government or characterized by a “lack of judicially discoverable and manageable standards.” Neither condition applied in *Baker*, and the Court ordered the state to reapportion.

Warren and Brennan had won Stewart’s vote in *Baker* by convincing him that the decision was narrow, but the “Reapportionment Revolution” was a cataclysm. Justice Whittaker had a nervous breakdown over *Baker*, recused himself from the case, and retired. Frankfurter suffered a stroke a week after delivering his masterful dissent, and resigned as well.⁵ Within two years, the Court formalized its “one person, one vote” standard (*Gray v. Sanders* 372 U.S. 368, 1963) and applied it to congressional districts (*Wesberry v. Sanders* 376 U.S. 1, 1964) and both houses of state legislatures (*Reynolds v. Sims* 377 U.S. 533, 1964). Fighting *Reynolds*, Senate Minority Leader Everett Dirksen brought the country within one vote of a Constitutional Convention. Reflecting in retirement on a tenure punctuated by *Brown* and

⁵Frankfurter argued that the definition of the level of equipopulation was a fundamentally political question. Drawing examples from England and the early American states, he argued moreover that it had admitted political solutions. (*Baker v. Carr* 369 U.S. 186, 1962, p. 301-310) Second he argued with almost uniquely pertinent originalist sources that the 14th Amendment was not intended to grant Congress power to regulate the suffrage within the states. This argument used the fact that the Joint Committee on the Reconstruction that drafted the 14th Amendment was also charged with restoring the southern states to the Union. This resulted in debates that directly touched on the concern. He argued further that the Union consisted of three categories of states: (i) Union states that approved the Amendment, (ii) Southern states readmitted to the Union under the Joint Committee, and (iii) states that have joined since the Amendment’s ratification. The large majority of these states that did not apportion representatives equally on population, established a legal precedent for unequal apportionment congenital with the Amendment itself. Finally, Frankfurter assessed whether the Court had any say over a state matter (capricious state action), and found that it did not.

Miranda, Chief Justice Warren called *Baker* the most consequential decision of his Court. (Smith 2014)

The Court eliminated malapportionment in just two years, but the success was not unalloyed. With *Kirkpatrick v. Preisler* (394 U.S. 526, 1969), the Court clarified that the “as nearly as practicable” population equality of *Gray* and *Wesberry* was less than 10%. In the majority opinion for *Karcher v. Daggett* (462 U.S. 725, 1983), Brennan rejected even sub-percent levels of inequality and indeed any *de minimis* threshold. This had two ill effects. The ‘supremacy’ of the equipopulation requirement was absolute. In his dissent to *Baker*, Frankfurter argued that the balance of equipopulation with other principles was the very core of the political problem to be avoided by the Court. The Court ruled at one extreme. Other countries require looser, 10-40% level of equality, which they balance with other representational principles: minority representation, preservation of existing subdivisions, etc. (Stephanopoulos 2013) But one need not appeal to foreign examples: when Congress debated reasserting its authority over districts in 1967, the proposed threshold on deviations was similarly just 10%. (Cong. Quarterly 1968) Further, the Court’s absolute adherence to equipopulation required fine manipulation of district lines. It instigated a skyrocketing level of split communities (Altman 1998), and “normalized” the fingery divisions now familiar in district maps. The second problem was that after the Court signalled its assumption of responsibility for districting, it has struggled to deliver beyond apportionment, and Congress has failed to contribute.

G.2.2. *Political Gerrymandering*. Political gerrymandering, the manipulation of district lines for partisan advantage, has of late received significant national attention. (Burns and Martin 2017; McIntee 2016; Oliver 2017) Rodden and Chen have conclusively demonstrated that demographic effects like Democrats’ tendency to cluster in cities currently favors Republicans. Nevertheless, they and others have also shown that a number of states’ maps show persistent, significant, and growing seat advantages for partisan mapmakers. I reproduce and elaborate both of these effects in Section 5. In Pennsylvania, Maryland, and North Carolina, the party controlling the legislature enacted maps for the 114th Congress that yield seats shares for that party significantly larger than those seen in the compactness-based simulations. Beyond seat shares, political gerrymandering has been widely demonstrated to reduce trust in government and voter turn-out. (Stephanopoulos 2013)

The harms perceived in political gerrymandering are thus real, and the Court since *Davis v. Bandemer* (478 U.S. 109, 1986) has held them to be justiciable. However, it has failed to enunciate a standard for adjudicating it, and therefore declined to provide relief. This is a curious position, since Brennan’s criterion for justiciability in *Baker* was the very existence of a “clear and discoverable standard” for the question at hand. The Court in *Davis* set a high bar for such a standard: plaintiffs must show intentional discrimination and actual discriminatory effect that persistently frustrates a group’s efforts to influence the political process as a whole.

In *Davis* and each subsequent case before the Court, a slender majority of justices has tendered proposals or held out hope, and in each case a slender majority has rejected every standard or justiciability itself. In his dissent (in judgment) to *Davis*, Powell wrote that standards from *Reynolds* ought to be adopted to political gerrymandering. “The most important of these factors are the shapes of voting districts and adherence to established political subdivision boundaries.”

With *Vieth v. Jubelirer* in 2004 (541 U.S. 267, 2004), four justices argued in favor of relief according to three standards. Stevens wrote that the “totality of the circumstances” approach used in racial gerrymandering cases had proven manageable and could be applied to

political gerrymandering. The argument is that racial cases provide an “alternate universe” where gerrymandering is regulated. Racial gerrymandering is unconstitutional under the 15th Amendment and regulated under the 1965 Voting Rights Act (as amended in 1970, 1975, 1982, and 2006). ([US Code §52 1965](#)) Confronted with this clear charge, the Court has enunciated consistent “traditional districting principles,” of compactness, contiguity, and respect for existing political subdivisions. The avowed stumbling block in political gerrymandering cases had not been the harm or the jurisdiction but the standard. The racial gerrymandering cases were proof positive of a standard. Stevens pointedly remarked, “the plurality does not argue that the judicially manageable standards that have been used to adjudicate racial gerrymandering claims would not be equally manageable in political gerrymandering cases.” (324)

Souter proposed that a plaintiff who was a member of an identifiable group could make a successful complaint by demonstrating that the group had suffered intentional harm from a plan enacted in disregard of traditional principles. As usual, the principles itemized were “contiguity, compactness, respect for political subdivisions, and conformity with geographic features.” Plaintiffs would have to show that plans that better-respected those principles were possible. (347-351)

For his part, Breyer penned a meditation beginning with “We the People” and seeking “workable form of government that is . . . basically democratic.” He ended by declaiming “*unjustified* entrenchment . . . purely the result of partisan manipulation and not other factors.” (356, 360, italics original)

Kennedy rejected each of these standards, and went out of his way to reject compactness: “even those criteria that might seem promising at the outset (e.g., contiguity and compactness) are not altogether sound as independent judicial standards for measuring a burden on representational rights. They [...] would unavoidably have significant political effect, whether intended or not.” He nevertheless declined to foreclose hope that a standard could eventually emerge.

Two other themes emerge in the Court’s deliberations in *Vieth*. The first is that a comfortable majority (the plurality, with Kennedy and Breyer) expressly endorsed politics as a “generally permissible” component of districting. (307) The second theme is the majority’s insistence on a pithy standard. In the plurality opinion, Scalia criticised Powell’s “flabby” approach which Kennedy echoed, calling for a “limited and precise rationale.”

Two years after *Vieth*, the Court in [League of United Latin American Citizens v. Perry](#) (548 U.S. 399, 2006) again failed to identify a standard, but Stevens (joined by Breyer) and Souter (joined by Ginsburg) expressed some interest in developing a standard on partisan symmetry.

The Court’s entry into congressional districting in the Reapportionment Revolution had enormous, immediate, and overwhelmingly positive impact on fair representation in America. Yet its legacy in political gerrymandering has ultimately mirrored Congress’ – a broad recognition of the harm, paired with a toothless consensus over the use of compactness as a component of fair districting laws. Powell’s prophecy for *Davis* has been fulfilled: it has signalled a “‘constitutional green light’ to would-be gerrymanderers” while simultaneously “inviting further litigation.”

What is still required – for Congress or the Court – is a “limited and precise” constraint on Congressional districting. Can compactness fill that need?

APPENDIX H. COMPACTNESS AS A LEGAL STANDARD

The legal history reviewed in Appendix G shows that both Congress and the Court have the potential to eliminate political gerrymandering. Both have recognized compactness as a

part of the solution. Section 5 argued that various notions of compactness line up with each other, and that implementing them results in consistent practical effects. Put boldly, there is only one compactness; it is a meaningful standard. Yet the House districts in Figure I.1 suggest even without the math, that current House districts are not uniformly compact. Why has compactness failed to gain traction?

Although American history suggests that Congress is at least as well suited to districting reform as the courts, the modern debate has centered on the judiciary. Three broad arguments stand out from that debate, as defenses of the status quo or critiques of compactness:

- (1) districting is a legitimate and even quintessentially political activity,
- (2) equations are an inadequate palette for expressing political communities, and
- (3) the selection of one definition of compactness over another amounts to a political choice.

The first argument – that districting is a political activity – is broadly agreed by the Court. In her concurrence (in judgment) to *Davis*, O'Connor rejected the Court's grounds and wrote that districting is a "political question in the truest sense." (*Davis v. Bandemer* 478 U.S. 109, 1986, p. 145) As remarked above, a majority affirmed this position in *Vieth*. (p. 307) The initial weakness of this argument is that the same defense was marshalled in support of malapportioned districts. The Reapportionment Revolution also recalibrated urban and rural power, and it is the nature of democracy to limit manipulation of the ballot box.

Moreover, inverting the question – asking whether politicians in practice use their authority to express an idealized 'social value' – makes a defense of the status quo less credible. The national parties' activities and avowed motives are partisan, not political, and do not create or elevate cognizable political communities. Unlike town or school district borders, the shifting lines of congressional (let alone state legislator) districts are hardly a standard 'source of identity.' Only around half of Americans know the political party of their representatives in Congress (Pew Research Center 2014), let alone his or her name or the district lines. In a defense of incumbent protections in districting, Nathaniel Persily presents some anecdotes of earnest legislators. He writes that through the districting process, "they create service relationships between representatives and constituents that fit into larger public policy program." (Persily 2002, p. 32) But his argument is stuck in reverse: in a Democracy this is precisely what is supposed to be what is decided in the ballot box – not the other way around.

The Republican State Leadership Committee's REDistricting MAjority Project (REDMAP) forwards explicitly partisan goals, which it has executed successfully. Democrats have signalled less-than-beneficent intent to compete on the same terms. (Burns and Martin 2017) In *Cooper v. Harris*, North Carolina Republicans even claimed partisan intent as a defense. And even less-noble justifications like "winner's bonuses" as for patronage fall flat in the context of elections: entrenchment between elections is anathema to Democracy. The practice is not in short, so high-minded as the principle that the justices appear to defend.

Several authors criticize automation as inadequately nuanced and human. Since it's impossible to subsume competing conceptions of public interests, communities, and identities in a single pithy metric, any effort is misplaced. Altman writes, "automated redistricting is significantly more complicated than gerrymandering" because the latter only requires "the maximization of one simple goal. Optimal redistricting may involve many simultaneous, complicated, and conflicting goals." Capturing the "social value" of "subtle patterns of community" in an equation or automated procedure is chasing leprechauns. (Altman 1997, pp. 82, 112) The geometric approach that I have presented is guilty as charged: it does not seek to encode every human interaction and adjacency. The fault of Altman's argument is that

gerrymandering is presently real and pernicious. Progress is possible without perfection; we ought not wallow in a swamp for the impossibility of reaching the end of the rainbow.

The final common critique of automation is that naming an algorithm is naming a winner. In past papers, the meat of this argument has been that choosing metric or algorithm was simply a re-labelled political choice. Altman wrote, “the idea that automated redistricting is not inherently objective seems both correct and unavoidable.” As cited above, Kennedy wrote in his equivocation for *Vieth* that contiguity and compactness “would unavoidably have significant political effect, whether intended or not.” Since I have shown that a broad array of metrics yields consistent seat shares, the political choice is more constrained than one might have naïvely believed.⁶ So it is true that the choice of an algorithm will affect electoral outcomes, but the fact that rules result in winners and losers hardly precludes their fairness or neutrality. The question then reduces to compactness’s suitability and fairness. While a criterion of partisan bias might generate different baseline seat shares, the historical and constitutional justification is weak. Compactness on the other hand, has remained central to the debate and woven through the law for over a century. It is a recognized, non-partisan criterion for the principle of geographic representation.

APPENDIX I. THE CONSISTENCY OF COMPACTNESS MEASURES ON HISTORICAL MAPS

In addition to understanding the political consistency of maps optimized for various measures it is interesting to know whether the various compactness scores or ranks agree on enacted maps. Asking whether a district is compact or not is an important practical question, quite distinct from the possibility of generating compact districts. It could well be that away from the “extrema” chosen by the automated procedures, the compactness measures would rank districts differently. This Section evaluates the consistency of compactness scores and ranks on congressional districts and finds that the spatial “measurements” agree well in practice.

This analysis requires data beyond that used for the main results. Populations are drawn from the 1990, 2000, and 2010 Decennial Censuses. (US Census Bureau 2016) Additional, block-group (1990) and block-level (2000, 2010) geometries are also needed to calculate population distributions. (US Census Bureau 2012b,c, 2013b)

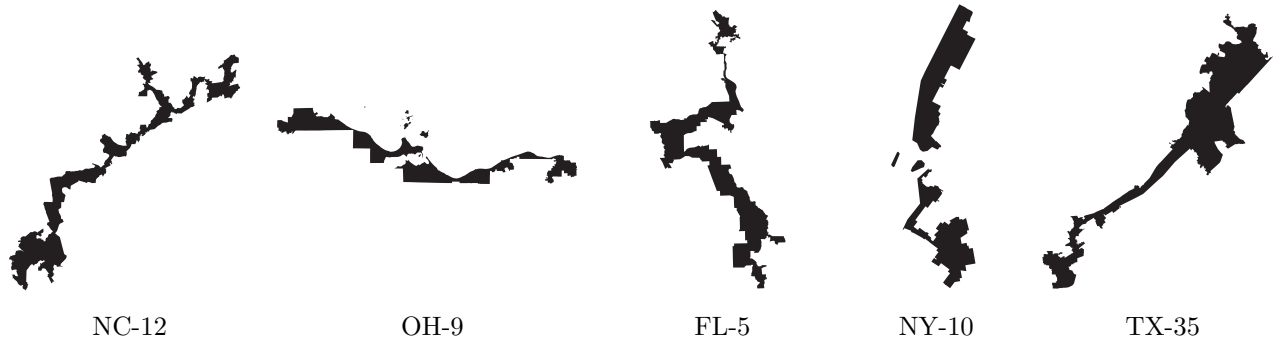
I then turn to the population of maps enacted in the last three districting cycles, in states with more than one representative. I evaluate all of the compactness measures aside path fraction, which has specific technical issues. The power diagrams, split-line procedure, and areal and population distance algorithms (Section 2.2) do not yield scores, and are not included in this analysis. However, I do replace power diagrams with the average interpersonal distance, with the normalization from Section 2.1.9. This gives me 14 measures of compactness (see Figure 3), and 1284 enacted districts.

I perform a principal component analysis (PCA) of this population using Scikit-learn.⁷ (Pedregosa et al. 2011) The explained variance of the first component is 69% and that of the second component is 14%. Excluding axis ratio, which has dubious merit, these numbers

⁶The fact that they give consistent results in automation does not mean that they are equally effective as *constraints* on gerrymandering. That is, when manipulating lines with partisan intent, it may be easy to keep some compactness scores high and harder for others. This point underscores the strength of a process-based, instead of results-based solution.

⁷A PCA reduces the dimensionality of a collection of measurements; within a population, it collapses as much of the variation as possible on to a single axis, or component. Of the remaining variation, it collapses as much as possible onto a second axis, and so forth. Each successive component is orthogonal to earlier ones.

FIGURE I.1. Presented are the five districts of the 114th Congress scored least compact by the first component of the compactness principal component analysis (see Section 5). The North Carolina, Florida, and Texas districts have since been struck down.



change to 73% and 15%. Renormalizing the measures to have equal variance before the PCA, the loading on the first component falls to 60% (10% on the second).

The interpretation is thus again that despite the different shapes shown in Figure I.2, compactness in the context of districting reduces to a single concept: it is one-dimensional. The five districts of the 114th Congress that were least compact according to the first principal component are presented in Figure I.1. It is notable that of these five, three have already been overturned. The sixth least-compact district is Hawaii’s second, which encompasses the entire 1500-mile long island chain, save the urban core of Honolulu. That district shows that non-compact districts are not necessarily gerrymandered, but it is the exception that proves the rule.

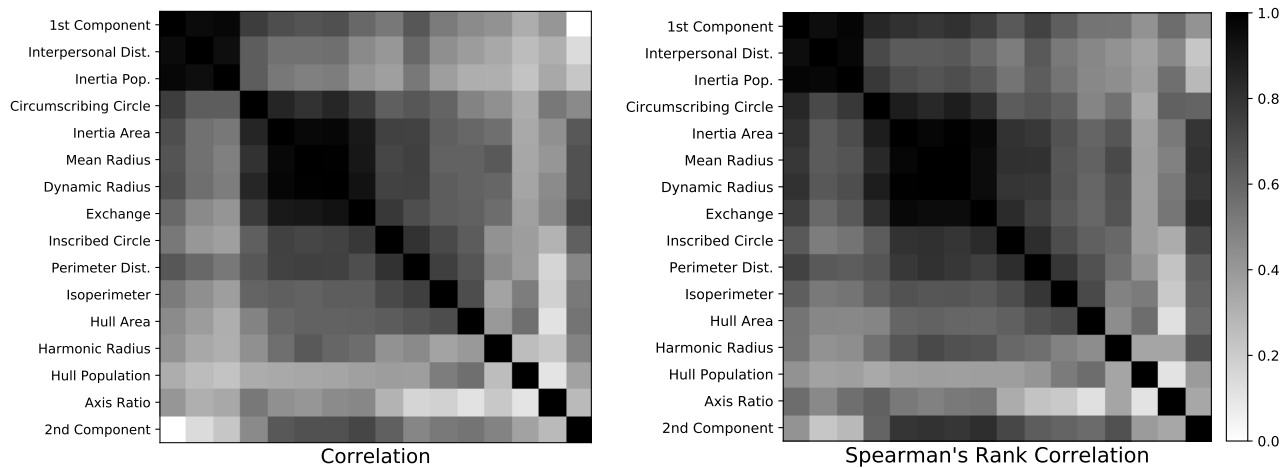
Figure I.2 presents the score and rank correlations among the fourteen measures and the PCA factors. Darker shading (closer to 1) represents higher correlation. As expected, all correlations are positive except between the two components of the PCA in the Spearman’s rank analysis. Axis ratio is notably less-correlated with the other measures. The remaining thirteen have fairly high score and rank correlations; a poor compactness score or rank by one definition generally predicts a poor score or rank on another definition. By construction, the first principal component is highly correlated with the other measures. It is a good proxy measure of compactness. The correlation with interpersonal distance is particularly high ($\rho = 0.95$); this means that power diagrams will result in good scores across measures. Although the PCA suggests that a large part of compactness is explained by a single axis, I have block-diagonalized the matrices to highlight the various “notions” of compactness.

This analysis suggests that the question of whether or not a real district is compact can meaningfully be answered: it is not just a question of the choice of ruler. From a practical perspective, the first principal component from the analysis represents a robust – though opaque – combined measure of compactness. Optimizing plans for interpersonal distance (or generating power diagrams) will produce districts that score well by all measures.

REFERENCES

An Act for the Apportionment of Representatives to Congress among the several States according to the ninth Census, 42d Congress, 2d session; 17 Stat. 28 (1872). [🔗](#)

FIGURE I.2. Correlations among measures of compactness, and with the first two principal components. The correlations are all positive except between the first two primary components in the Spearman's rank correlation, and so the absolute value is taken.



An act making an apportionment of Representatives in Congress among the several States under the tenth Census, 47th Congress, 1st session; 22 Stat. 5 (1882). [🔗](#)

An Act Making an apportionment of Representatives in Congress among the several States under the Twelfth Census, 56th Congress, 2d session; 31 Stat. 733 (1901). [🔗](#)

An Act For the apportionment of Representatives in Congress among the several States under the Thirteenth Census, 62d Congress, 1st session; 37 Stat. 13 (1911). [🔗](#)

Altman, M. 1997. "Is automation the answer: the computational complexity of automated redistricting." *Rutgers Computer and Law Technology Journal* **23** 81-142 (1997). [🔗](#)

Altman, M. 1998. "Traditional Districting Principles: Judicial Myths vs. Reality." *Social Science History* **22** (2): 159-200 (1998). DOI: [10.2307/1171534](#)

Altman, M., MacDonald, K., and McDonald, M. 2005. "From Crayons to Computers." *Social Science Computer Review* **23** (3): 334-346 (2005). DOI: [10.1177/0894439305275855](#)

Altman, M. and McDonald, M. 2011. "BARD: Better Automated Redistricting." *Journal of Statistical Software* **42** (1): 1-28 (2011). DOI: [10.18637/jss.v042.i04](#)

Baker v. Carr, 369 U.S. 186 (1962). [🔗](#)

Burns, A. and Martin, J. 2017. "Eric Holder to Lead Democrats' Attack on Republican Gerrymandering." *The New York Times* (January 11, 2017). [🔗](#)

Bush v. Vera, 517 U.S. 952 (1996). [🔗](#)

Chen, J. and Rodden, J. 2013. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures." *Quarterly Journal of Political Science* **8** (3): 239-269 (2013). DOI: [10.1561/100.00012033](#)

Chen, J. and Rodden, J. 2015. "Cutting Through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders." *Election Law Journal* **14** (4): 331-345 (December, 2015). [🔗](#)

Chen, J. and Cottrell, D. 2016. "Evaluating partisan gains from Congressional gerrymandering: Using computer simulations to estimate the effect of gerrymandering in the U.S. House." *Electoral Studies* **44** 329-340 (2016). DOI: [10.1016/j.electstud.2016.06.014](#)

- Cho, W. K. T. and Liu, Y. Y. 2016. "Toward a Talismanic Redistricting Tool: A Computational Method for Identifying Extreme Redistricting Plans." *Election Law Journal* **15** (4): 351-366 (December, 2016). DOI: [10.1089/elj.2016.0384](https://doi.org/10.1089/elj.2016.0384)
- Chou, C., Kimbrough, S., Sullivan-Fedock, J., Woodard, C. J., and Murphy, F. H. 2012. "Using Interactive Evolutionary Computation (IEC) with Validated Surrogate Fitness Functions for Redistricting," at *14th Annual Conference on Genetic and Evolutionary Computation*, Philadelphia, Pennsylvania, USA, 1071-1078 DOI: [10.1145/2330163.2330312](https://doi.org/10.1145/2330163.2330312)
- Chou, C., Kimbrough, S. O., Murphy, F. H., Sullivan-Fedock, J., and Woodard, C. J. 2014. "On Empirical Validation of Compactness Measures for Electoral Redistricting and Its Significance for Application of Models in the Social Sciences." *Social Science Computer Review* **32** (4): 534-543 (2014). DOI: [10.1177/0894439313484262](https://doi.org/10.1177/0894439313484262)
- Cong. Globe, 27th Cong., 2d sess. Vol 11, pp. 407 and 788-790 (agreement on the principle), 449-451 and 786-788 (doom), 790 (state strategies). (May 2, 1842; June 4 and 10, 1842) [🔗](#)
- Cong. Globe, 37th Cong., 2d sess. Vol 33, pp. 2910-2912. (June 24, 1862) [🔗](#)
- Cong. Rec., 56th Cong., 2d sess. Vol 34, pp. 605-606. (January 5, 1901) [🔗](#)
- Cong. Rec., 70th Cong., 2d sess. Vol 70, pp. 1496, 1499, 1584, 1602, 1604. (January 10-11, 1929) [🔗](#) The statute as enacted is [46 Stat. 26](#).
- Coooper v. Harris, Supreme Court 518 (2016). [🔗](#)
- Davis v. Bandemer, 478 U.S. 109 (1986). [🔗](#)
- Duque, J. C., Church, R. L., and Middleton, R. S. 2011. "The p-Regions Problem." *Geographical Analysis* **43** (1): 104-126 (2011). DOI: [10.1111/j.1538-4632.2010.00810.x](https://doi.org/10.1111/j.1538-4632.2010.00810.x)
- Duque, J. C., Dev, B., Betancourt, A., and Franco, J. L. 2011. *ClusterPy: Library of spatially constrained clustering algorithms, Version 0.9.9.*, Colombia: RiSE-group (Research in Spatial Economics). EAFIT University.. [🔗](#)
- Duque, J. C., Anselin, L., and Rey, S. J. 2012. "The Max-p-Regions Problem." *Journal of Regional Science* **52** (3): 397-419 (2012). DOI: [10.1111/j.1467-9787.2011.00743.x](https://doi.org/10.1111/j.1467-9787.2011.00743.x)
- Feo, T. A. and Resende, M. G C. 1989. "A probabilistic heuristic for a computationally difficult set covering problem." *Operations Research Letters* **8** (2): 67-71 (1989). DOI: [10.1016/0167-6377\(89\)90002-3](https://doi.org/10.1016/0167-6377(89)90002-3)
- Fifield, B., Higgins, M., Imai, K., and Tarr, A. 2017. *A New Automated Redistricting Simulator Using Markov Chain Monte Carlo*. (Princeton University; Princeton, NJ). [🔗](#)
- Fryer, R. G., Jr. and Holden, R. 2011. "Measuring the Compactness of Political Districting Plans." *The Journal of Law and Economics* **54** (3): 493-535 (2011). DOI: [10.1086/661511](https://doi.org/10.1086/661511)
- Glover, F. 1989. "Tabu SearchPart I." *ORSA Journal on Computing* **1** (3): 190-206 (1989). DOI: [10.1287/ijoc.1.3.190](https://doi.org/10.1287/ijoc.1.3.190)
- Gray v. Sanders, 372 U.S. 368 (1963). [🔗](#)
- Karcher v. Daggett, 462 U.S. 725 (1983). [🔗](#)
- Kimbrough, S. O., Kuo, A., Chuin, L. H., Murphy, F. H., and Wood, D. H. 2011. "Solution Pluralism and Metaheuristics," at *MIC 2011: The IX Metaheuristics International Conference*, Udine, Italy, [🔗](#)
- Kirkpatrick v. Preisler, 394 U.S. 526 (1969). [🔗](#)
- Kurland, P. and Lerner, R. 2000. *The Founders' Constitution*, Vol. 2, Article 1, Section 4, Clause 1 University of Chicago Press. [🔗](#) See in particular the documents on the ratifying conventions of [Pennsylvania](#), [Massachussetts](#), [Virginia](#), [New York](#), and [North Carolina](#), and [Federalist 10](#), [56](#), and [57](#) (Madison), and [59](#) (Hamilton).
- League of United Latin American Citizens v. Perry, 548 U.S. 399 (2006). [🔗](#)

- Li, W., Goodchild, M. F., and Church, R. 2013. "An efficient measure of compactness for two-dimensional shapes and its application in regionalization problems." *International Journal of Geographical Information Science* **27** (6): 1227-1250 (2013). DOI: [10.1080/13658816.2012.752093](https://doi.org/10.1080/13658816.2012.752093)
- Li, W., Church, R. L., and Goodchild, M. F. 2014. "An extendable heuristic framework to solve the p-compact-regions problem for urban economic modeling." *Computers, Environment and Urban Systems* **43** 1-13 (2014). DOI: [10.1016/j.compenvurbsys.2013.10.002](https://doi.org/10.1016/j.compenvurbsys.2013.10.002)
- Liu, Y. Y., Cho, W. K. T., and Wang, S. 2016. "PEAR: a massively parallel evolutionary computation approach for political redistricting optimization and analysis." *Swarm and Evolutionary Computation* **30** 78-92 (2016). DOI: [10.1016/j.swevo.2016.04.004](https://doi.org/10.1016/j.swevo.2016.04.004)
- McIntee, M. 2016. "Student Asks Obama About Cynicism and Gets a 10 Minute Rant That Nails It." *YouTube Video* (April 28, 2016). [🔗](#)
- Miller v. Johnson, 515 U.S. 900 (1995). [🔗](#)
- Oliver, J. 2017. "Gerrymandering: Last Week Tonight with John Oliver." *YouTube Video* (April 9, 2017). [🔗](#)
- Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., *et al.* 2011. "Scikit-learn: Machine Learning in Python." *Journal of Machine Learning Research* **12** 2825-2830 (2011).
- Persily, N. 2002. "In Defense of Foxes Guarding Henhouses: The Case for Judicial Acquiescence to Incumbent-Protecting Gerrymanders." *Harvard Law Review* **116** (2): 649-683 (2002). DOI: [10.2307/1342612](https://doi.org/10.2307/1342612)
- Rey, S. J. and Anselin, L. 2010. "PySAL: A Python Library of Spatial Analytical Methods." In *Handbook of Applied Spatial Analysis: Software Tools, Methods and Applications*. Berlin: Springer. [🔗](#)
- Reynolds v. Sims, 377 U.S. 533 (1964). [🔗](#)
- Schwartzberg, J. E. 1966. "Reapportionment, Gerrymanders, and the Notion of Compactness." *Minnesota Law Review* **50** 443-452 (1966).
- Shaw v. Reno, 509 U.S. 630 (1993). [🔗](#)
- Smith, J. D. 2014. *On Democracy's Doorstep*, New York, NY: Hill and Wang. [🔗](#)
- Spann, A., Kane, D., and Gulotta, D. 2007. "Electoral Redistricting with Moment of Inertia and Diminishing Halves Models." *UMAP Journal* **28** (3): 233 (2007). [🔗](#)
- Stephanopoulos, N. 2013. "Our Electoral Exceptionalism." *University of Chicago Law Review* **80** (769): (2013). [🔗](#)
- Vickrey, W. 1961. "On the Prevention of Gerrymandering." *Political Science Quarterly* **76** (1): 105-110 (1961). DOI: [10.2307/2145973](https://doi.org/10.2307/2145973)
- Vieth v. Jubelirer, 541 U.S. 267 (2004). [🔗](#)
- Weaver, J. B. and Hess, S. W. 1963. "A Procedure for Nonpartisan Districting: Development of Computer Techniques." *The Yale Law Journal* **73** (2): 288-308 (1963). DOI: [10.2307/794769](https://doi.org/10.2307/794769)
- Wesberry v. Sanders, 376 U.S. 1 (1964). [🔗](#)
- Wood v. Broom, 287 U.S. 1 (1932). [🔗](#)
- Zagarri, R. 1987. *The Politics of Size*, Ithaca: Cornell University Press, pp. 150-152.
- Cong. Quarterly 1968. *Congress Fails to Adopt House District Standards*, Washington DC: Congressional Quarterly, pp. 550-557. [🔗](#)
- Cong. Quarterly 1994. *Guide to US Elections*, Washington D.C.: Congressional Quarterly, pp. 932-970.
- Pew Research Center 2014. *GOP Has Midterm Engagement Advantage*. (Washington, DC), pp. 3. [🔗](#)

Republican State Leadership Committee, “The Redistricting Majority Project.” Accessed June 21, 2017. [🔗](#)

US Census Bureau, “2000 Block Shapefiles.” Updated March, 2012. Accessed December 12, 2017. <ftp://ftp2.census.gov/geo/tiger/TIGER2010/TABBLOCK/2000/>

US Census Bureau, “1990 Block Group Shapefiles.” Updated March 25, 2012. Accessed December 12, 2017. <ftp://ftp2.census.gov/geo/tiger/TIGER2010/TABBLOCK/2010/>

US Census Bureau, “1990 Block Group Shapefiles.” Updated June 19, 2013. Accessed December 12, 2017. [🔗](#)

US Census Bureau, “Decennial Census (2010, 2000, 1990).” 2016. [🔗](#) Data series has been updated since data were retrieved.

US Code §52, Subtitle I (1965). [🔗](#)

US Code Title 2, §2c (Amended 1967). [🔗](#) See the statute amending this section (81 Stat. 581), and [Cong. Rec. \(1929\)](#) for the history of the passage of the 1929 apportionment. The Apportionment Act of 1911 (37 Stat. 13) was the last to contain references to contiguity, compactness, and equality of population.